Principal Component Analysis Demystified

Caroline Walker

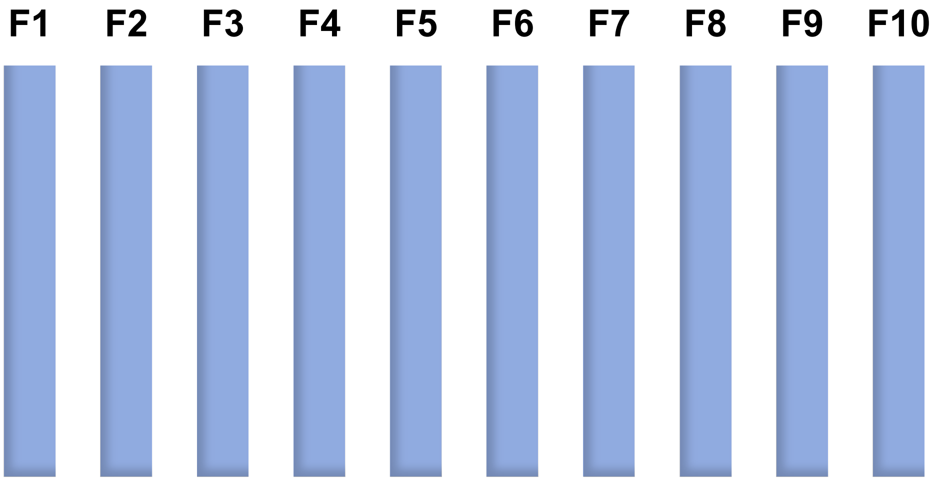
Warren Rogers LLC

Principal Component Analysis

Goal: Dimensionality Reduction

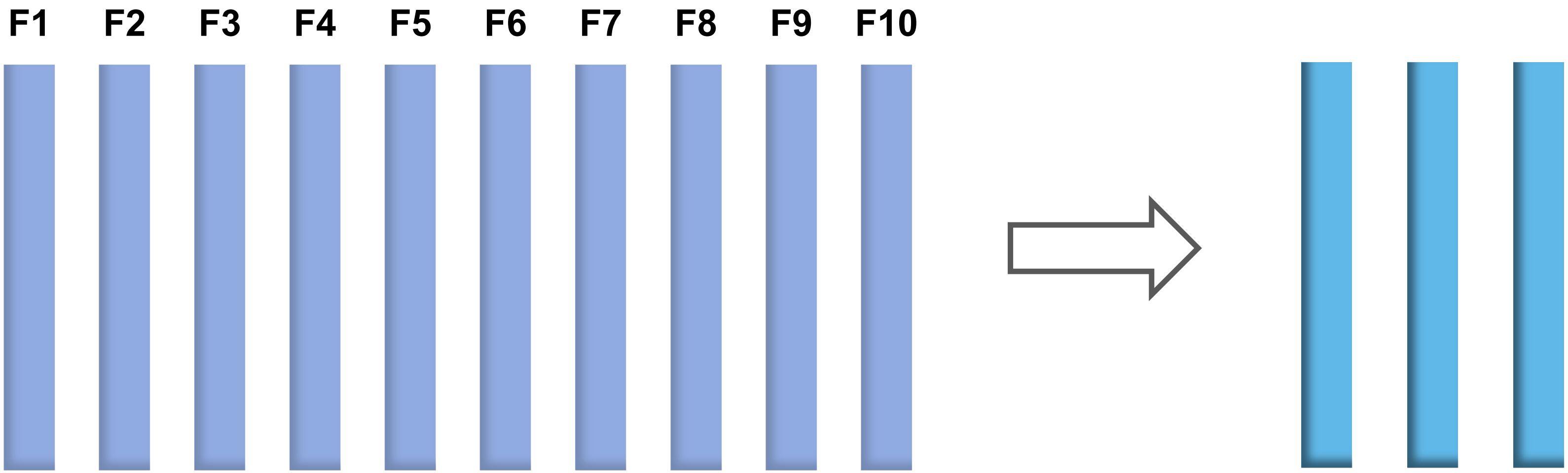
Principal Component Analysis

Goal: **Dimensionality Reduction**



Principal Component Analysis

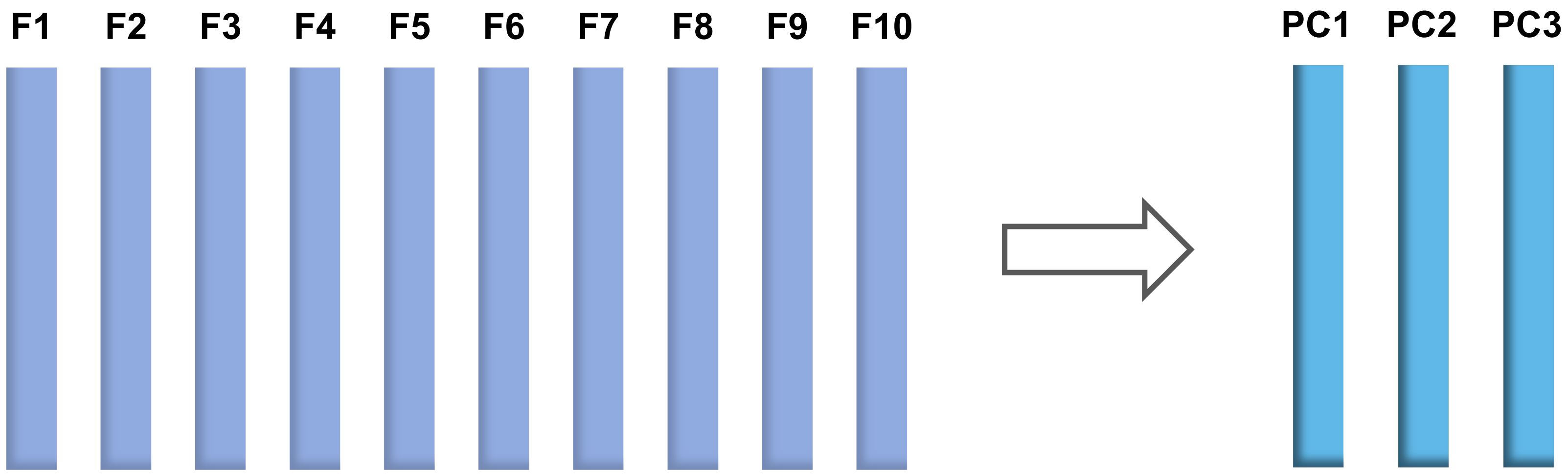
Goal: Dimensionality Reduction



#SASGF

Principal Component Analysis

Goal: Dimensionality Reduction



Principal Component Analysis

The new features…

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- Are linear transformations of the original features

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- Are linearly uncorrelated with each other

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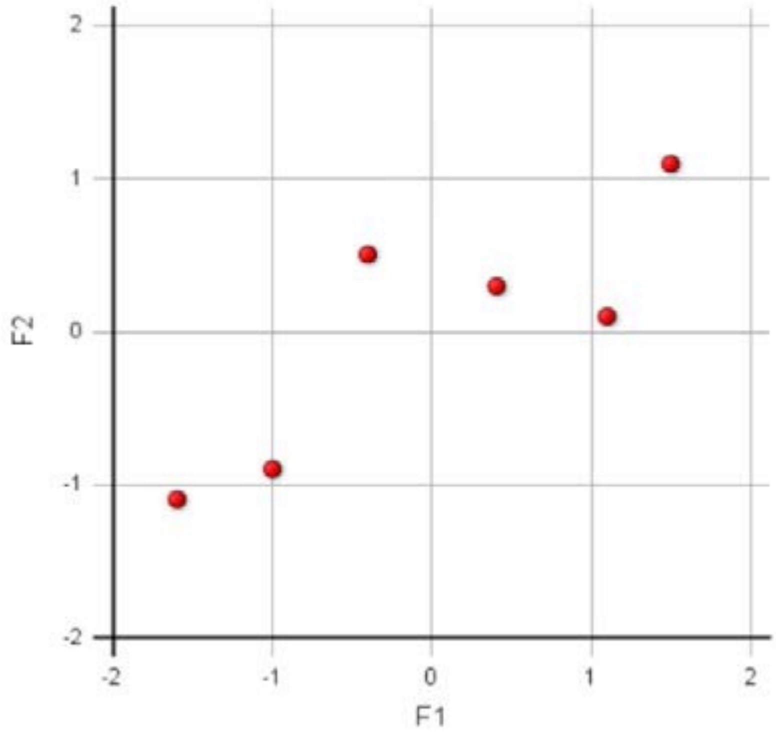
An Example in Two Dimensions



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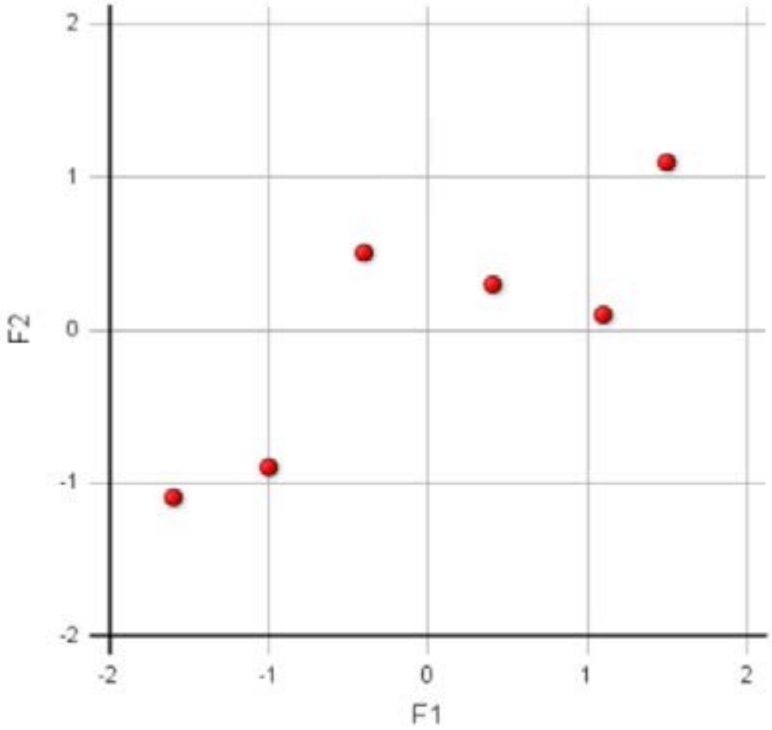
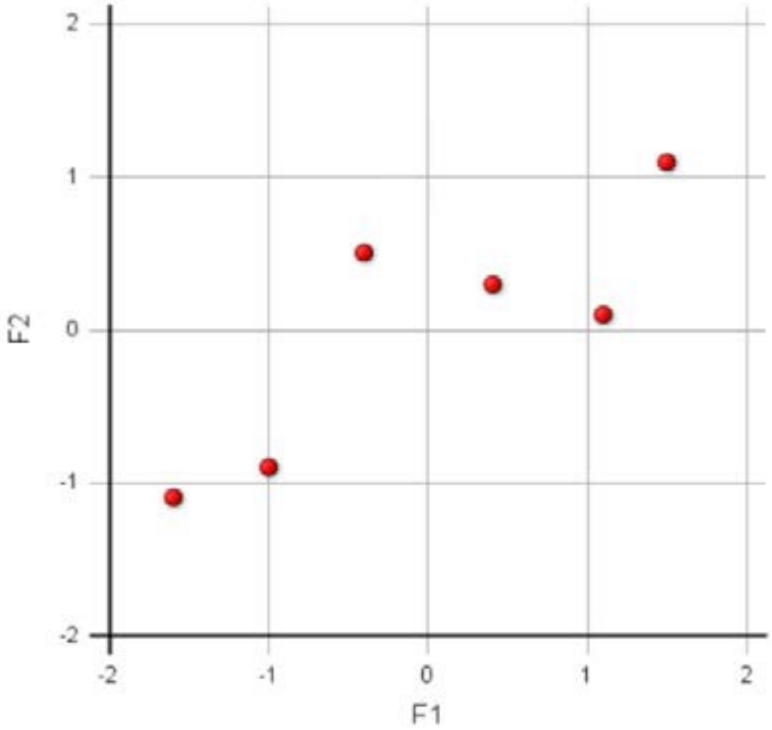


Feature	Variance
F1	1.468
F2	0.716

An Example in Two Dimensions



Feature	Variance
F1	1.468
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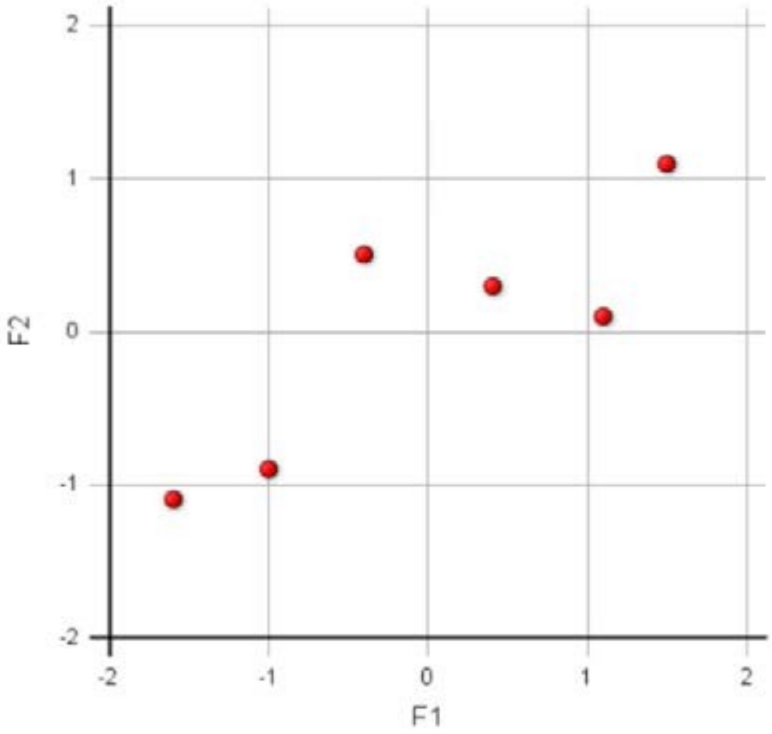
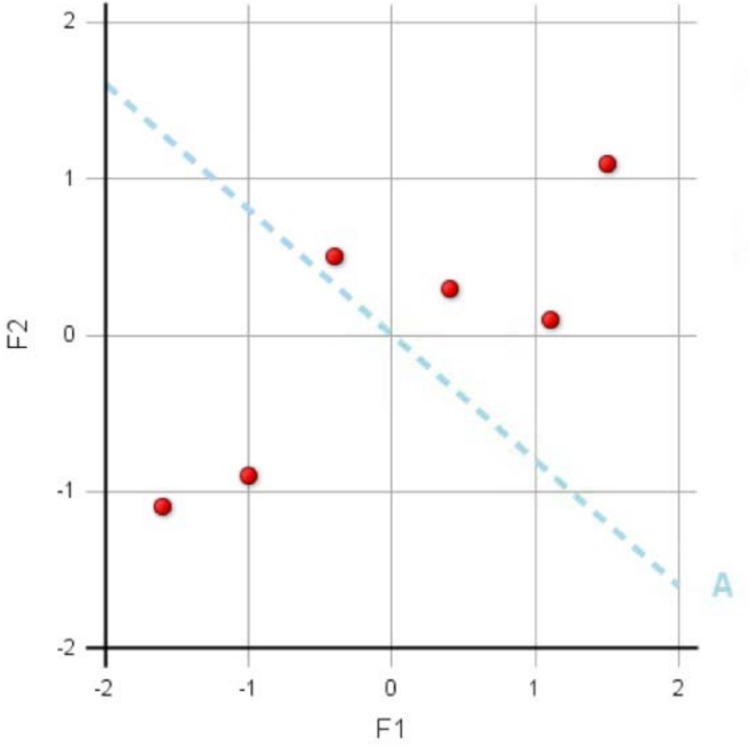
2.184

An Example in Two Dimensions



Evaluating a Possible New Feature

Feature A



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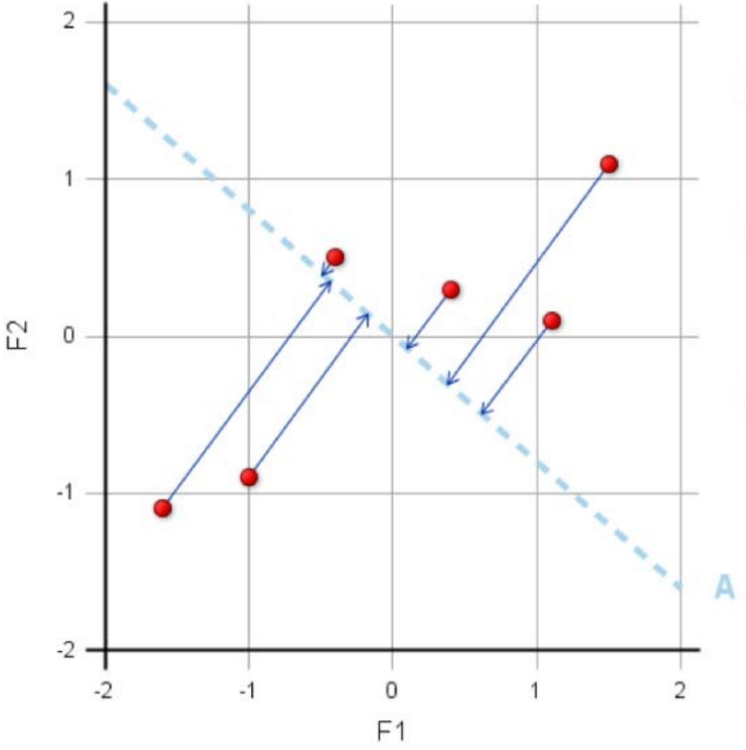

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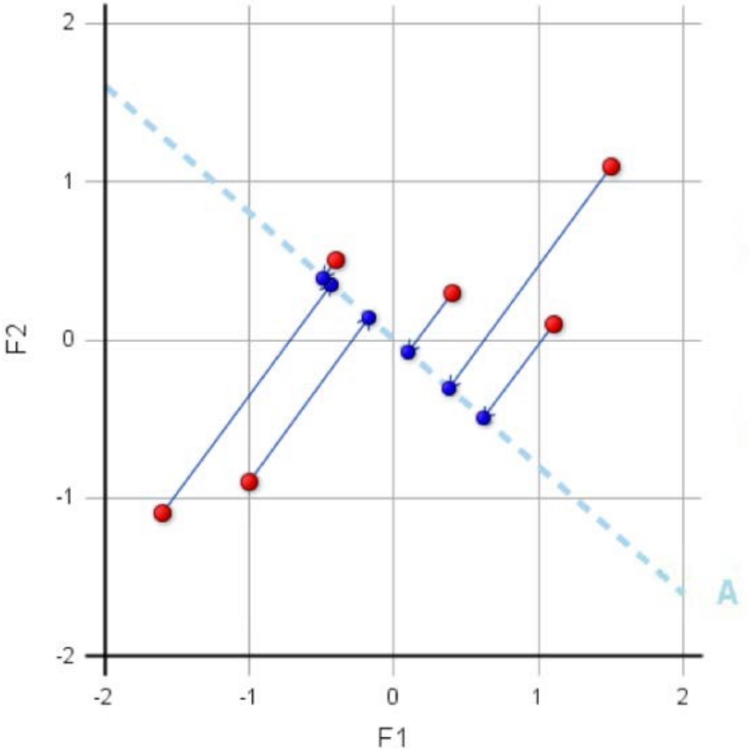

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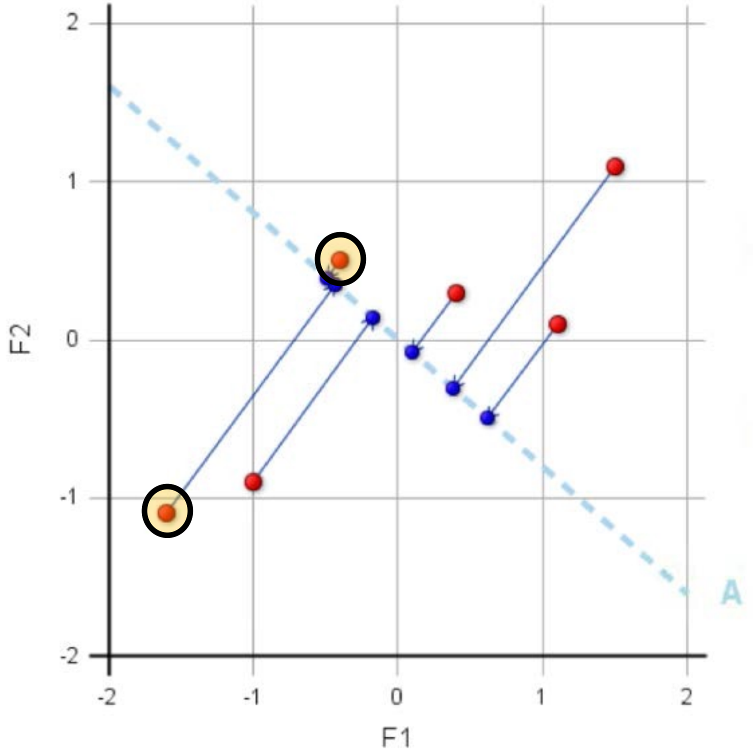

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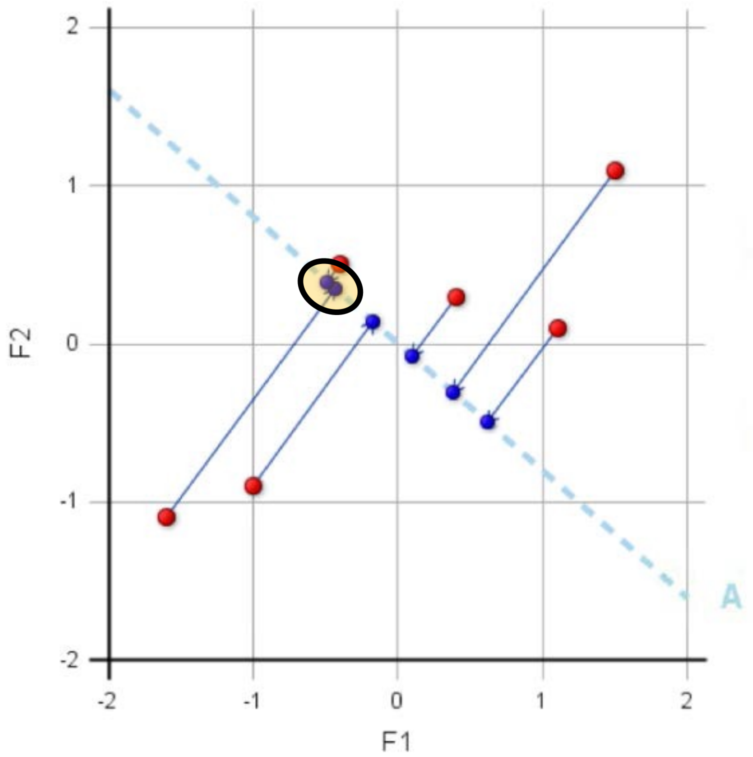

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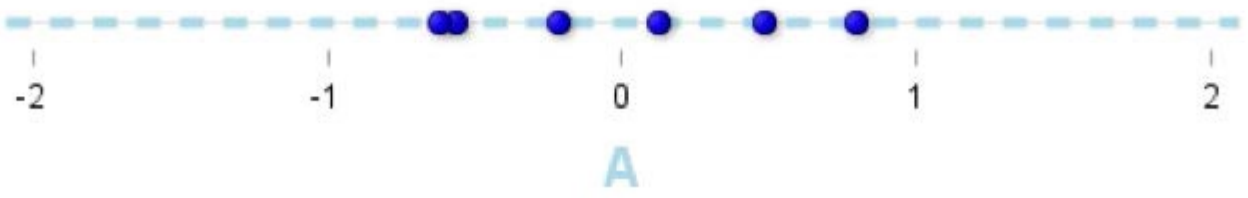

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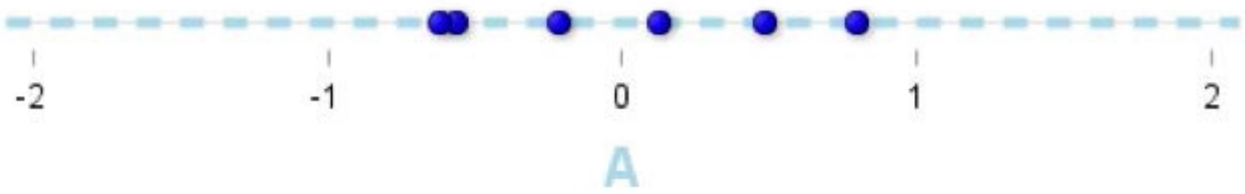
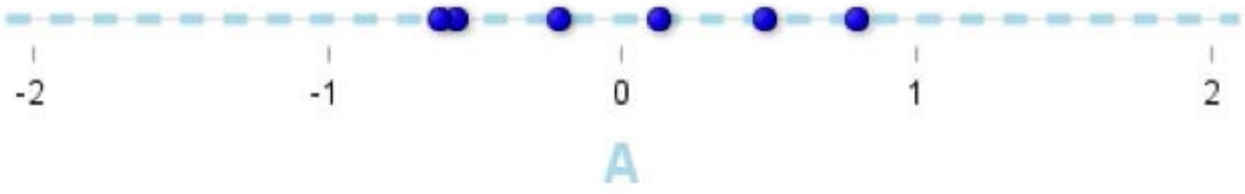
Feature A



Feature	Variance
A	0.328

Evaluating a Possible New Feature

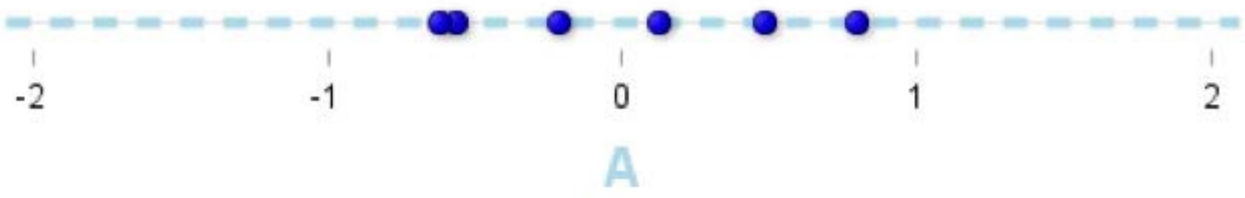
Feature A



Feature	Variance
A	**0.328**
F1	1.468
F2	0.716

Evaluating a Possible New Feature

Feature A



Feature	Variance
A	**0.328**
F1	1.468
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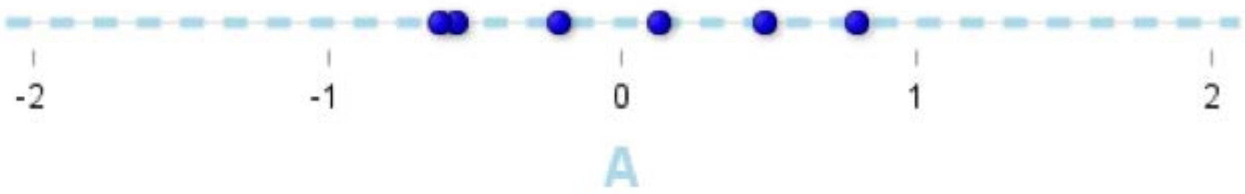
2.184

Evaluating a Possible New Feature

Feature A



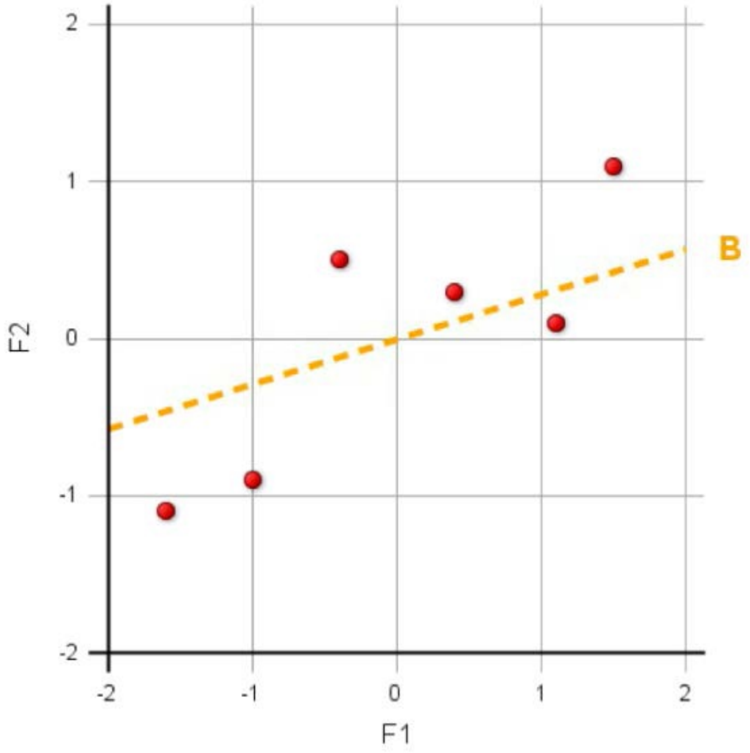
Feature	Variance
A	0.328
F1	1.468
F2	0.716

15%

2.184

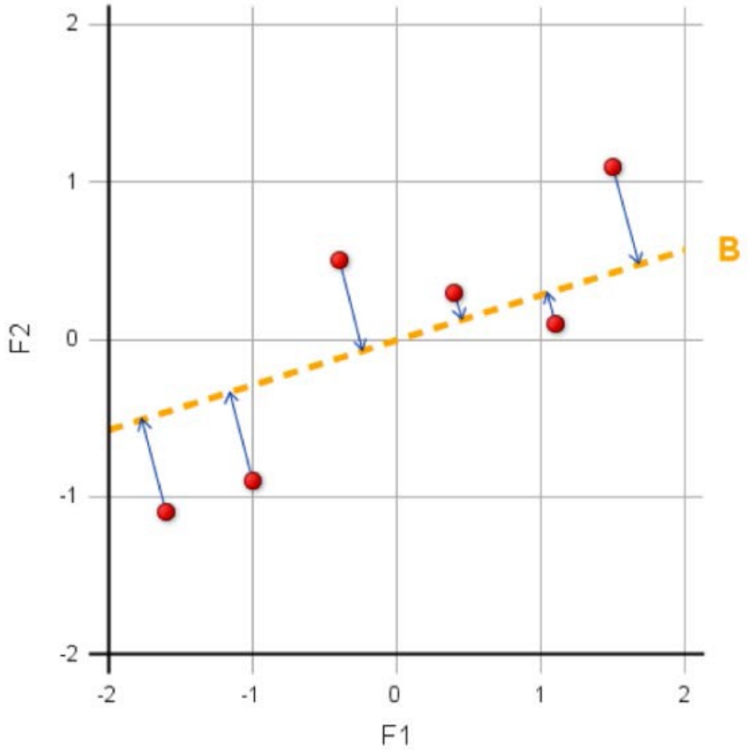
Evaluating a Possible New Feature

Feature B



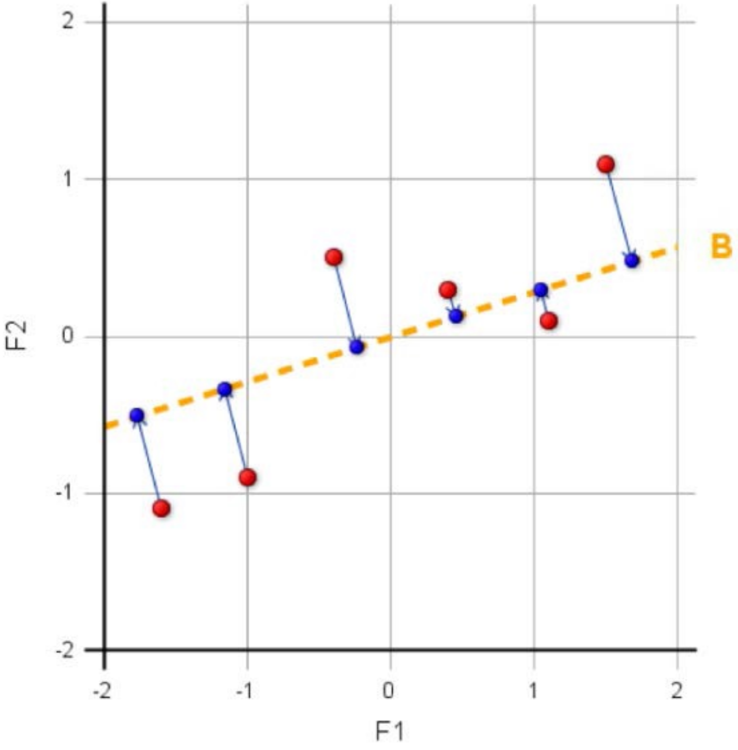
Evaluating a Possible New Feature

Feature B



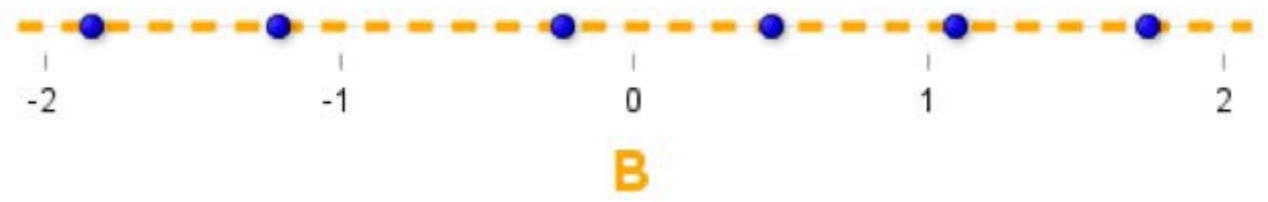
Evaluating a Possible New Feature

Feature B



Evaluating a Possible New Feature

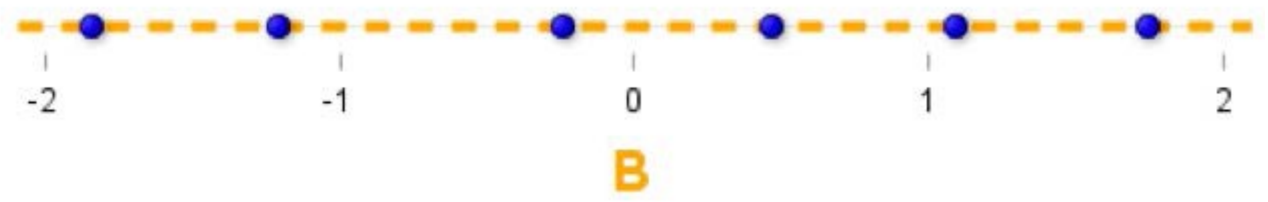
Feature B



Feature	Variance
B	1.869

Evaluating a Possible New Feature

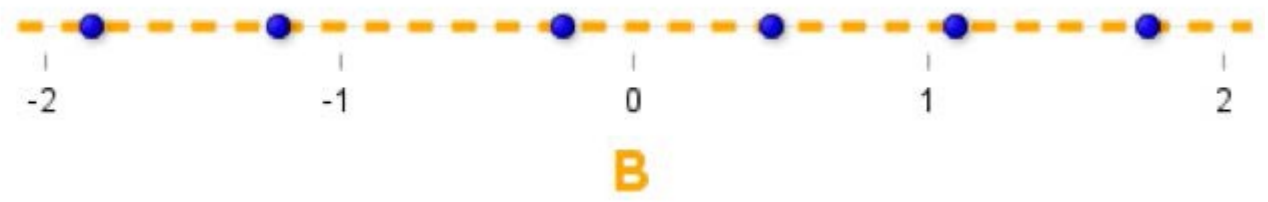
Feature B



Feature	Variance
B	1.869
F1	1.468
F2	0.716

Evaluating a Possible New Feature

Feature B



Feature	Variance
B	1.869
F1	1.468
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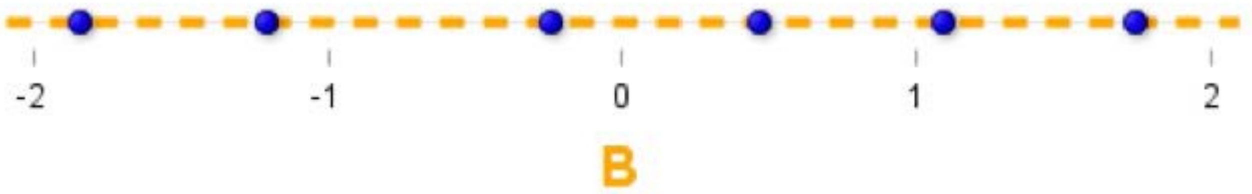
2.184

Evaluating a Possible New Feature

Feature B



Feature	Variance
B	1.869
F1	1.468
F2	0.716

86%

$\underbrace{1.468 + 0.716}_{}$ 2.184

Finding the Optimal Solution

Steps of PCA

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1. Calculate the covariance* matrix of the original feature set.

Finding the Optimal Solution

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2. Find the eigenvectors and eigenvalues of this covariance matrix.

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Steps of PCA

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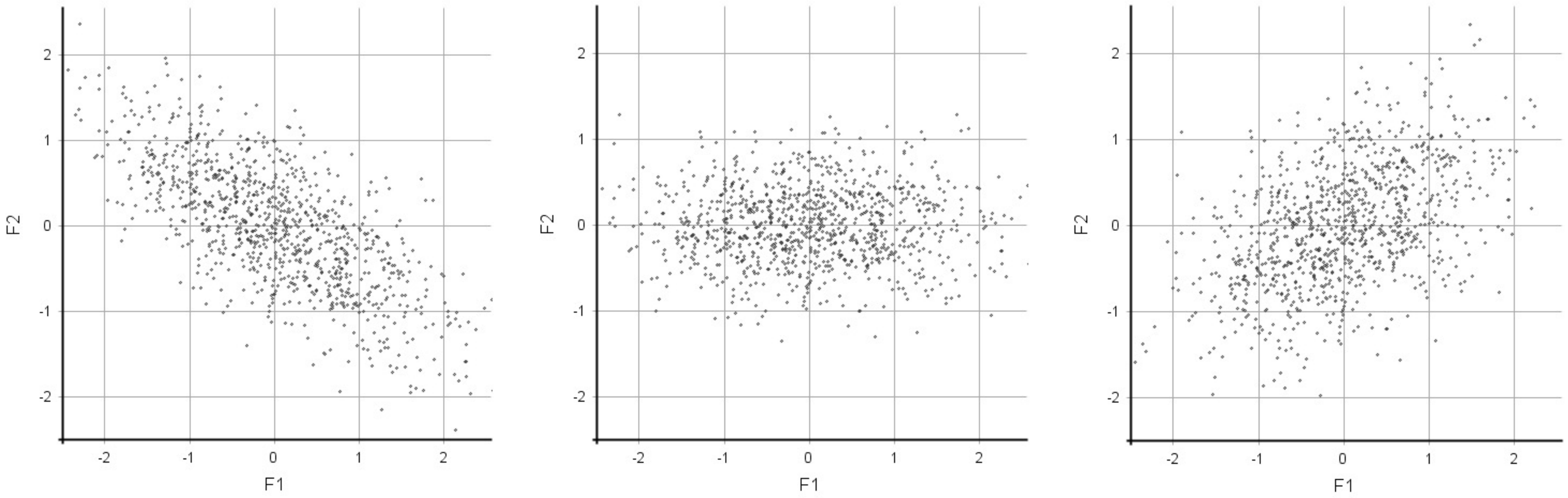
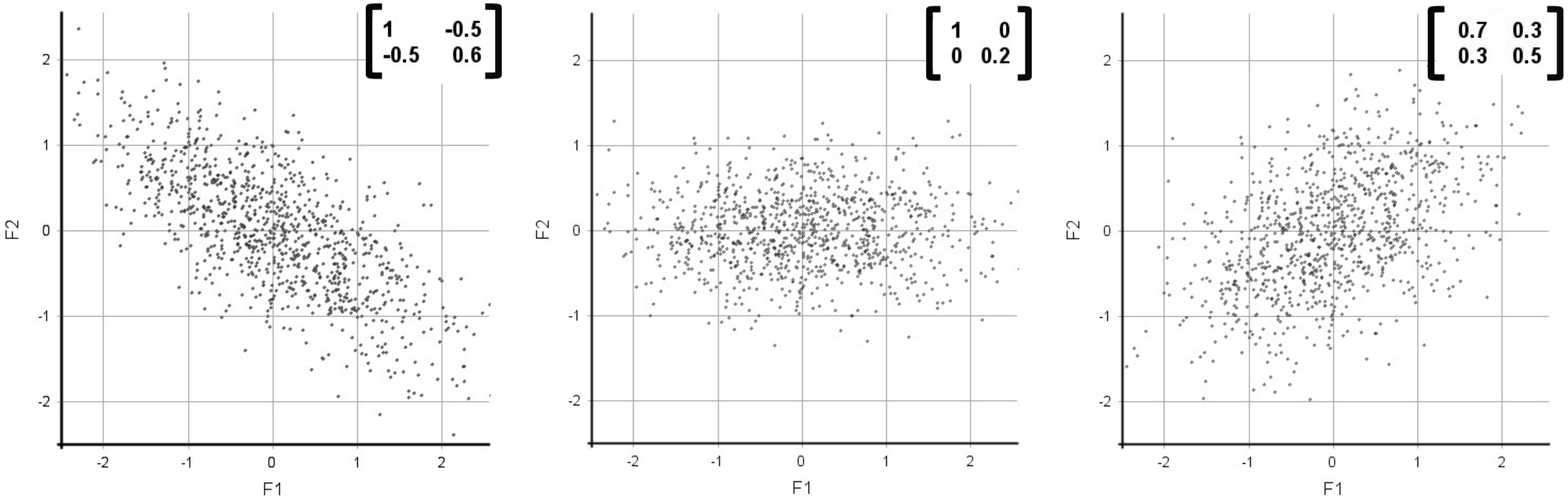
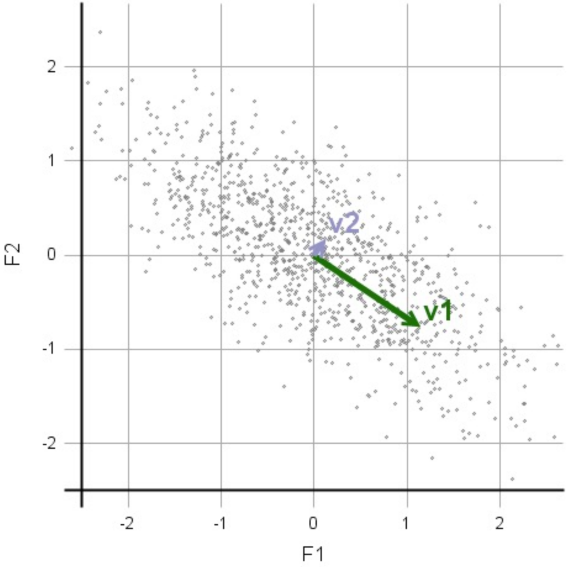
Finding the Optimal Solution

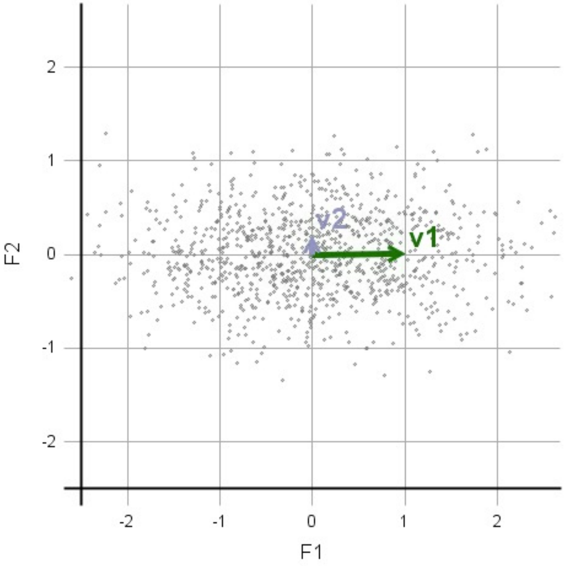
Steps of PCA

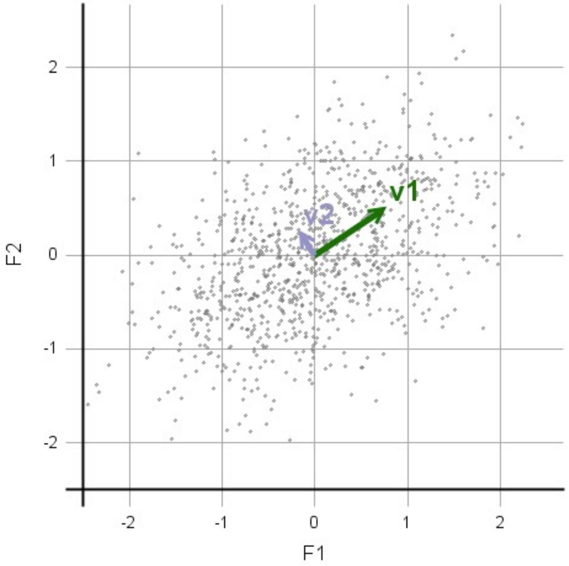
1. Calculate the covariance* matrix of the original feature set.
2. Find the eigenvectors and eigenvalues of this covariance matrix.
3. Order the eigenvectors according to the magnitude of their eigenvalues.
4. **The eigenvector v_1 will show the direction of maximum variance within the data set.**

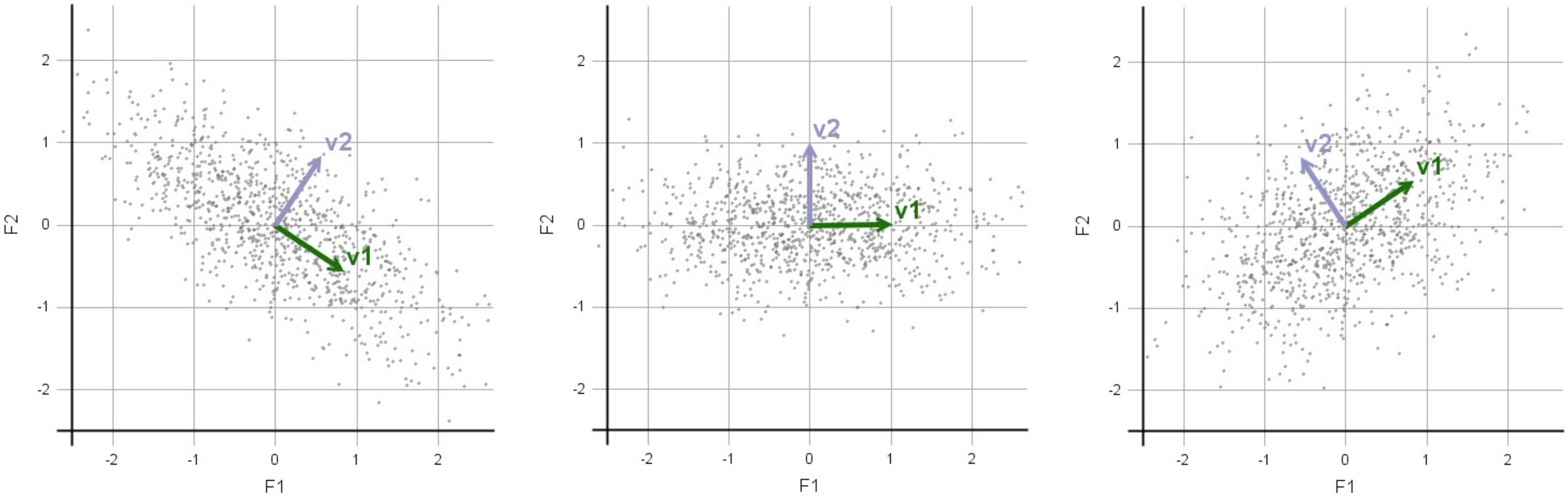

F2
F1
2
1
0
-1
-2
-2
-1
0
1
2


1 -0.5
-0.5 0.6
1 0
0 0.2
0.7 0.3
0.3 0.5
F1
F2
2
1
0
-1
-2


F2
F1
v1
v2
2
1
0
-1
-2
-2
-1
0
1
2

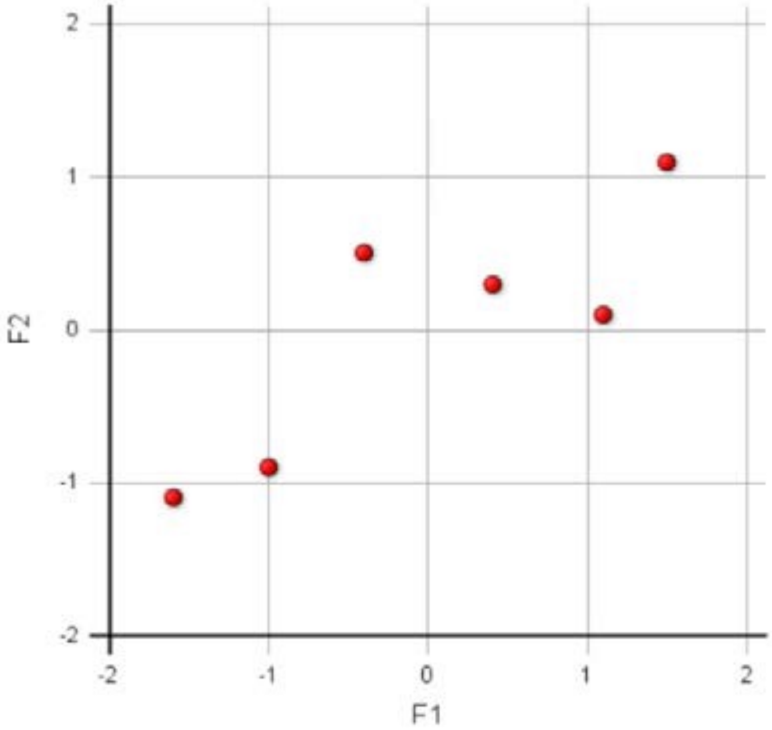

F2
F1
v1
v2
2
1
0
-1
-2
-2
-1
0
1
2


F2
F1
v1
v2
2
1
0
-1
-2
-2
-1
0
1
2



v2
v1
F1
F2
2
1
0
-1
-2

Principal Component Analysis



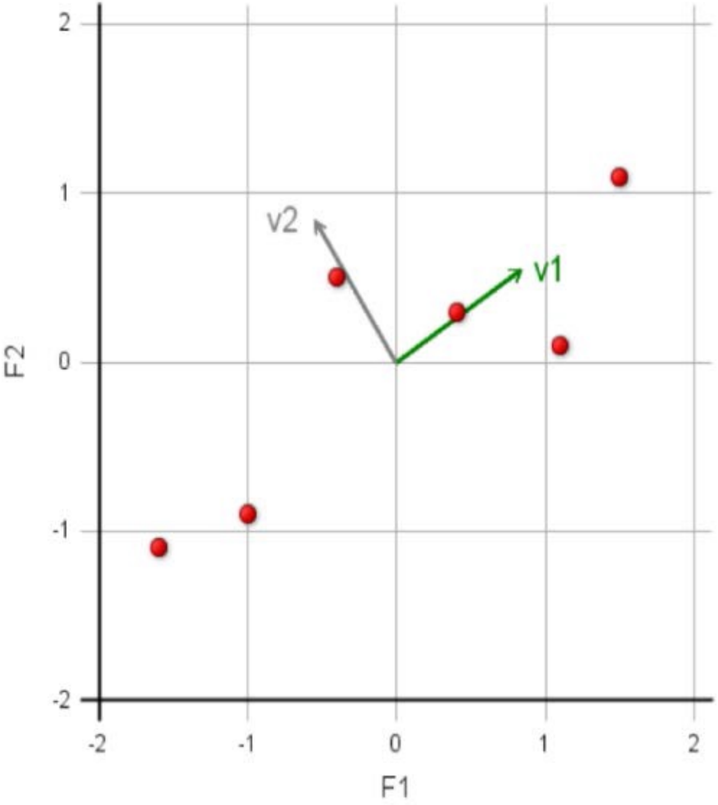
$$\begin{bmatrix} 1.468 & 0.868 \\ 0.868 & 0.716 \end{bmatrix}$$

Principal Component Analysis

$$\begin{bmatrix} 0.836 \\ 0.549 \end{bmatrix} \qquad \begin{bmatrix} -0.549 \\ 0.836 \end{bmatrix}$$

$$\lambda_1 = 2.038 \qquad \lambda_2 = 0.146$$

Principal Component Analysis



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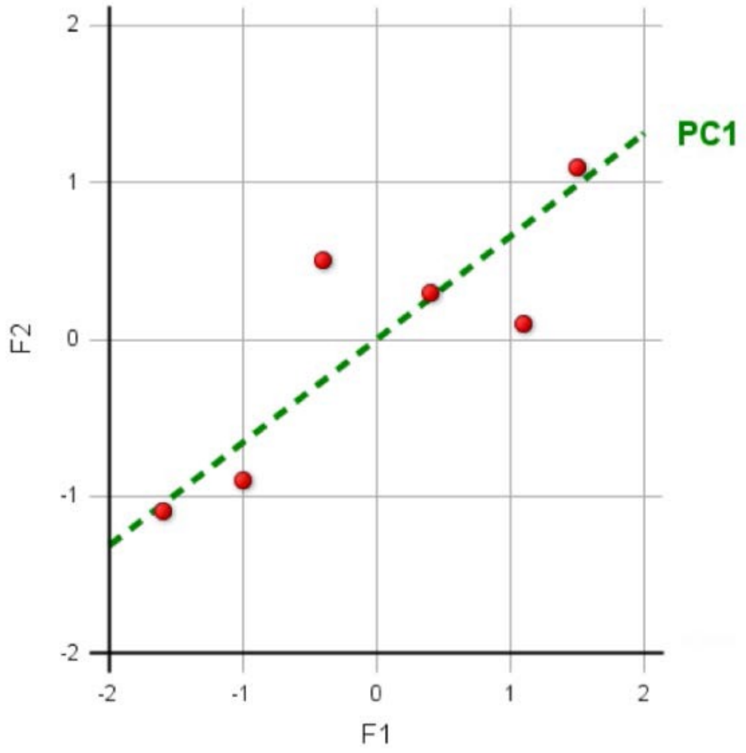

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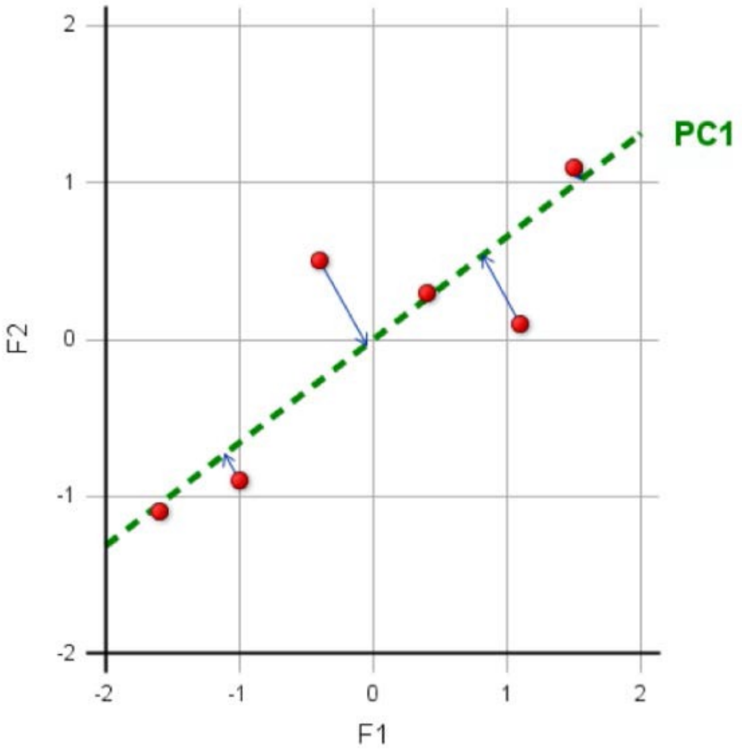

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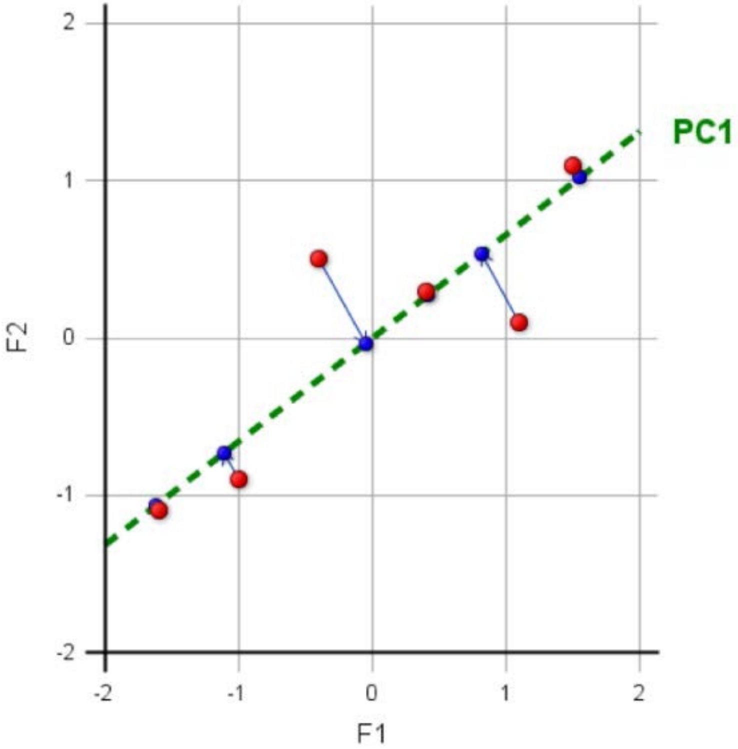

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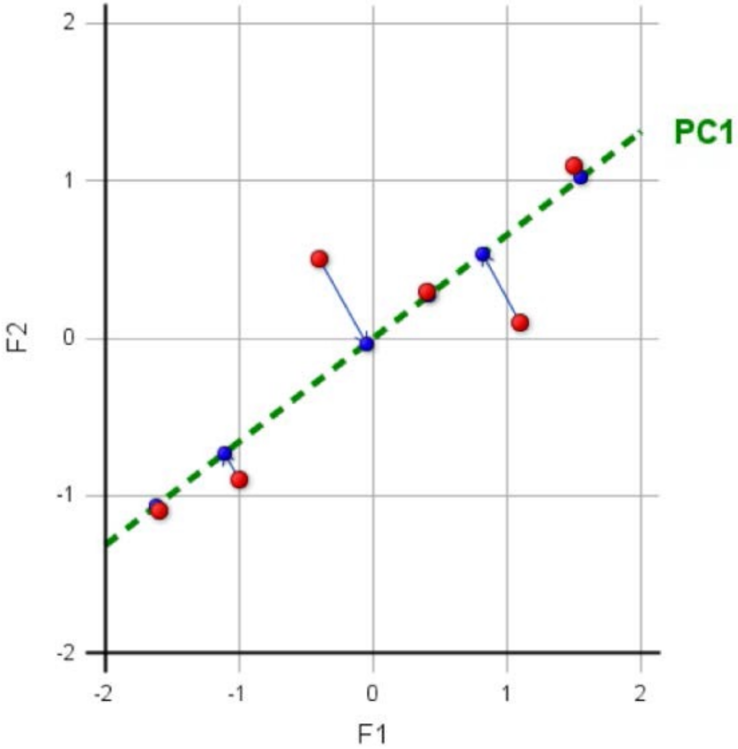
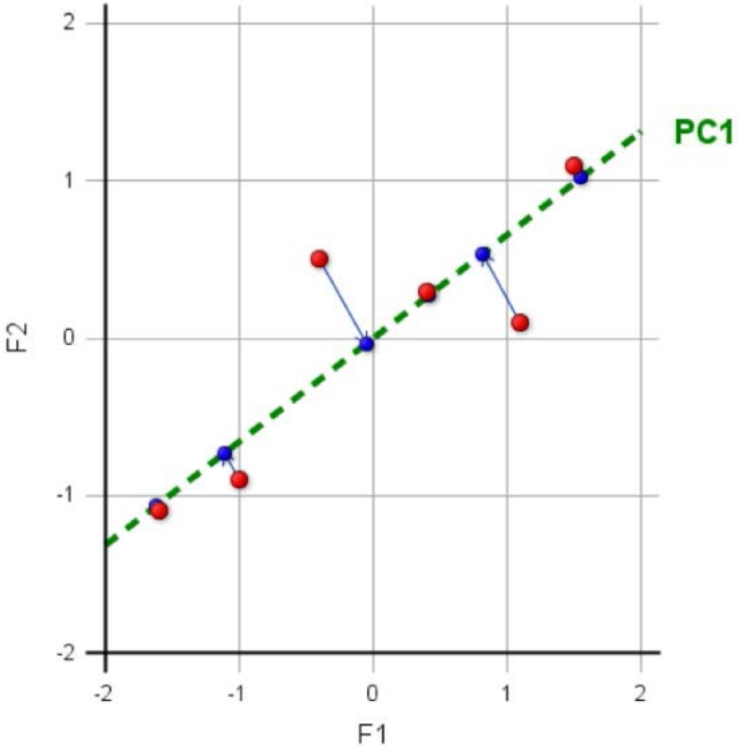


Feature	Variance
PC1	2.038

Principal Component Analysis



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F1	1.468
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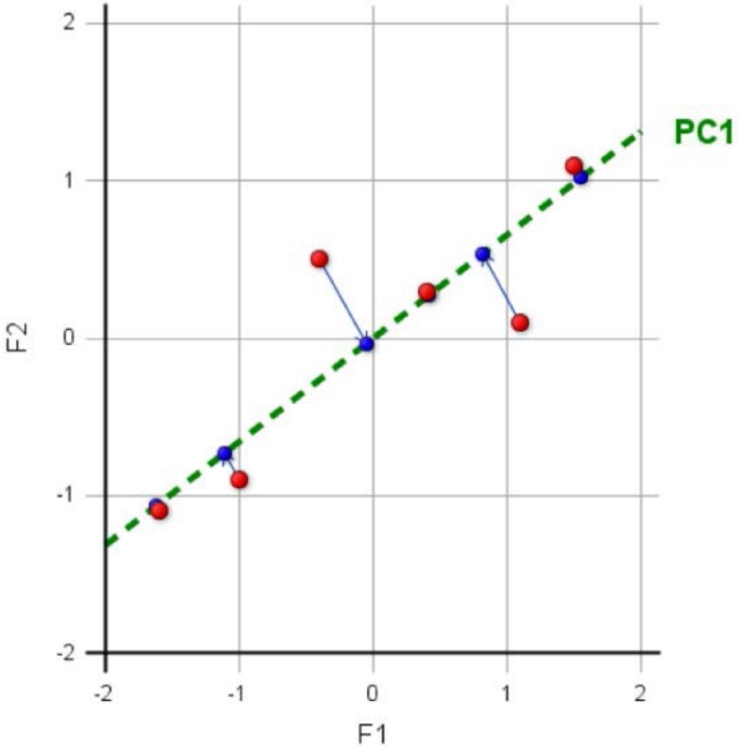
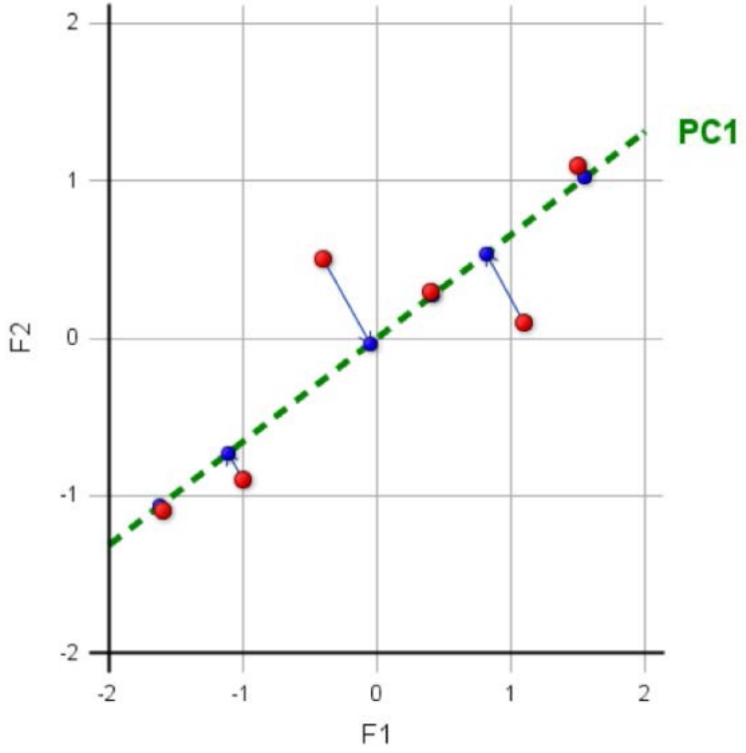


Feature	Variance
PC1	2.038
F1	1.468
F2	0.716

2.184

Principal Component Analysis



Feature	Variance
PC1	2.038
F1	1.468
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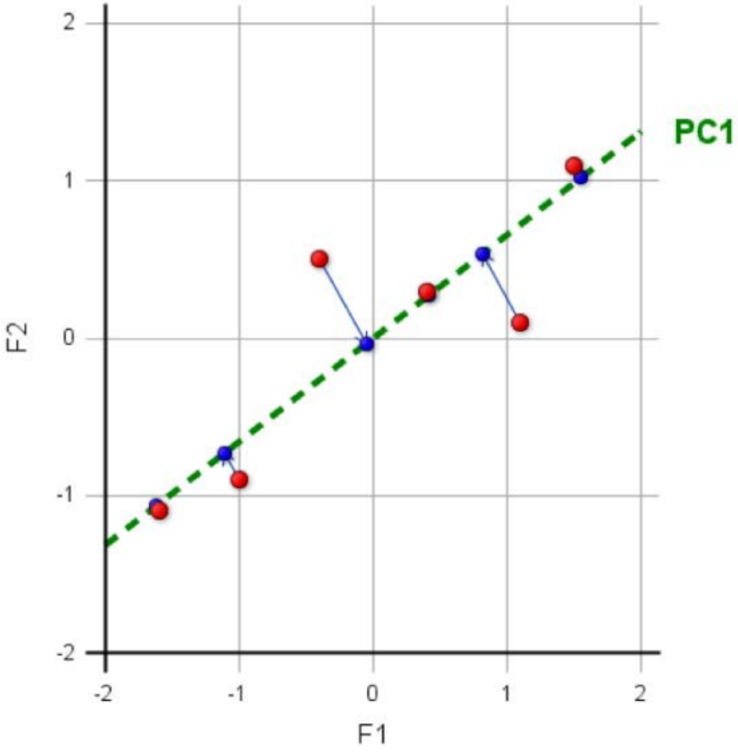
93%

2.184

Principal Component Analysis



Feature	Variance
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Principal Component Analysis

$$\begin{bmatrix} 0.836 \\ 0.549 \end{bmatrix} \qquad \begin{bmatrix} -0.549 \\ 0.836 \end{bmatrix}$$

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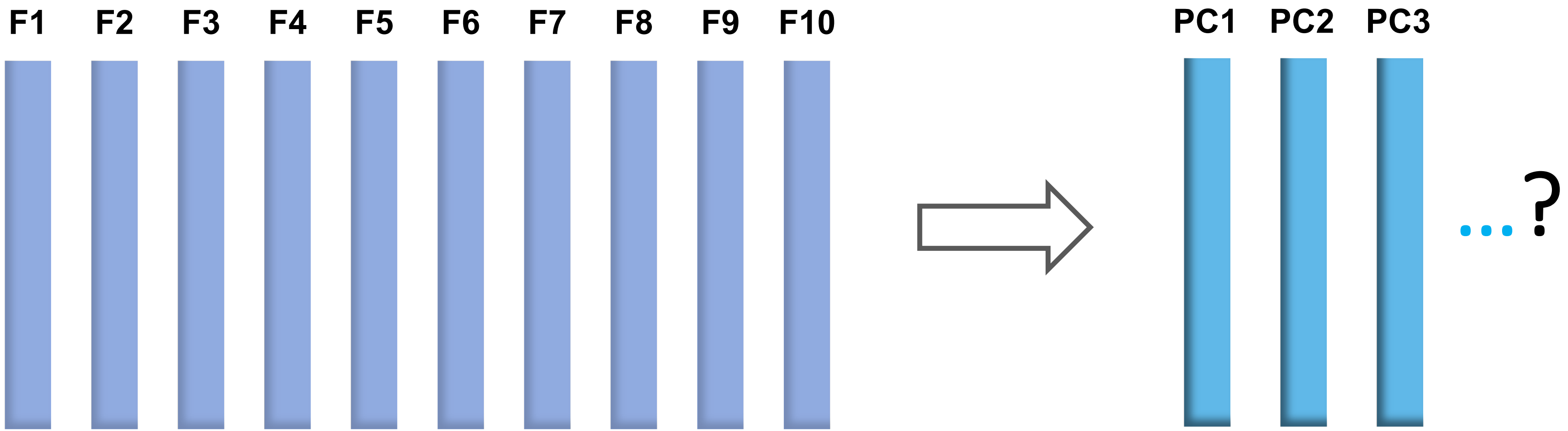
PCA in Higher Dimensions



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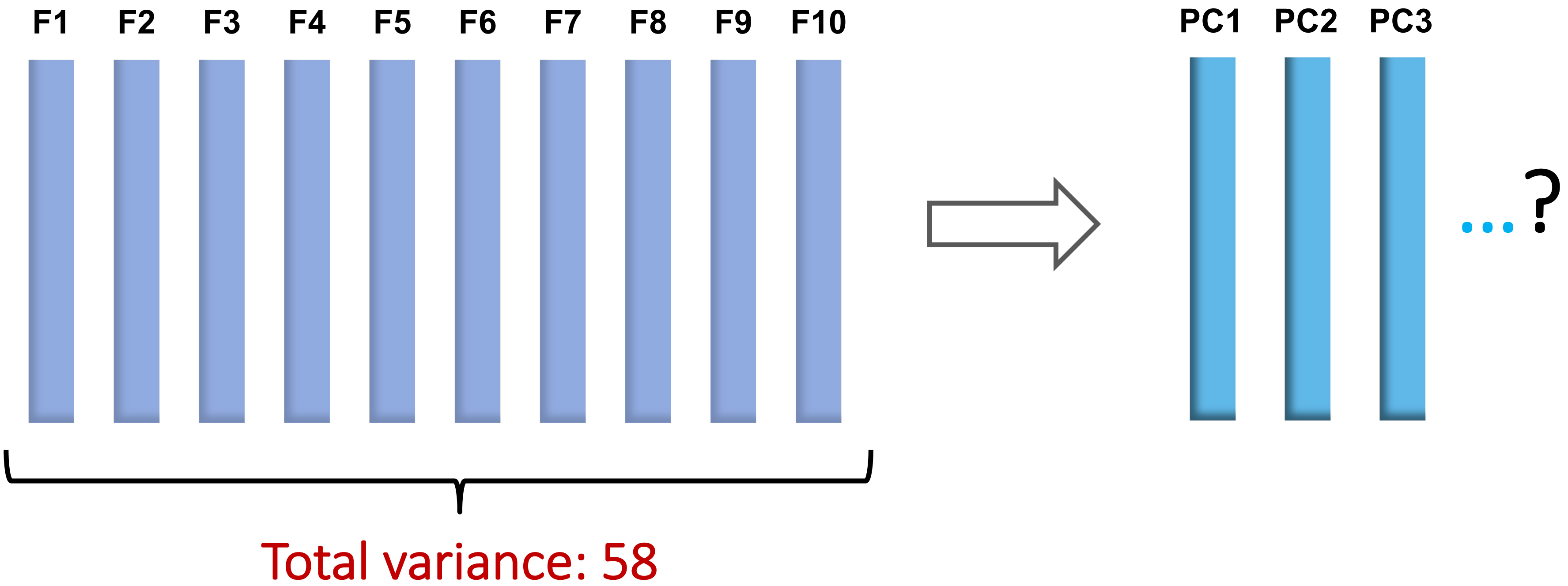

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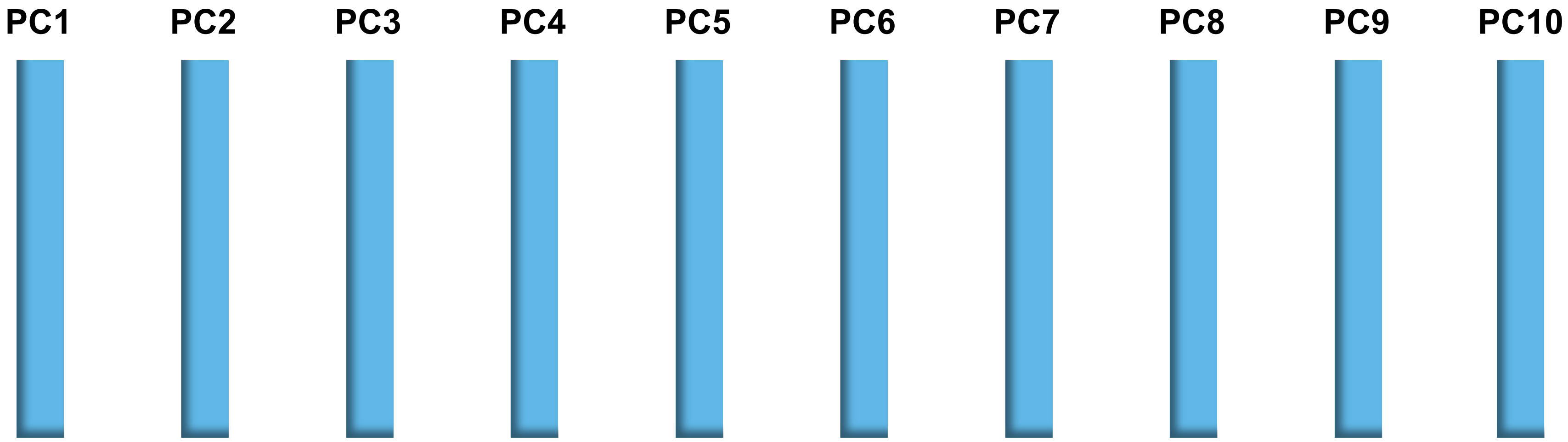

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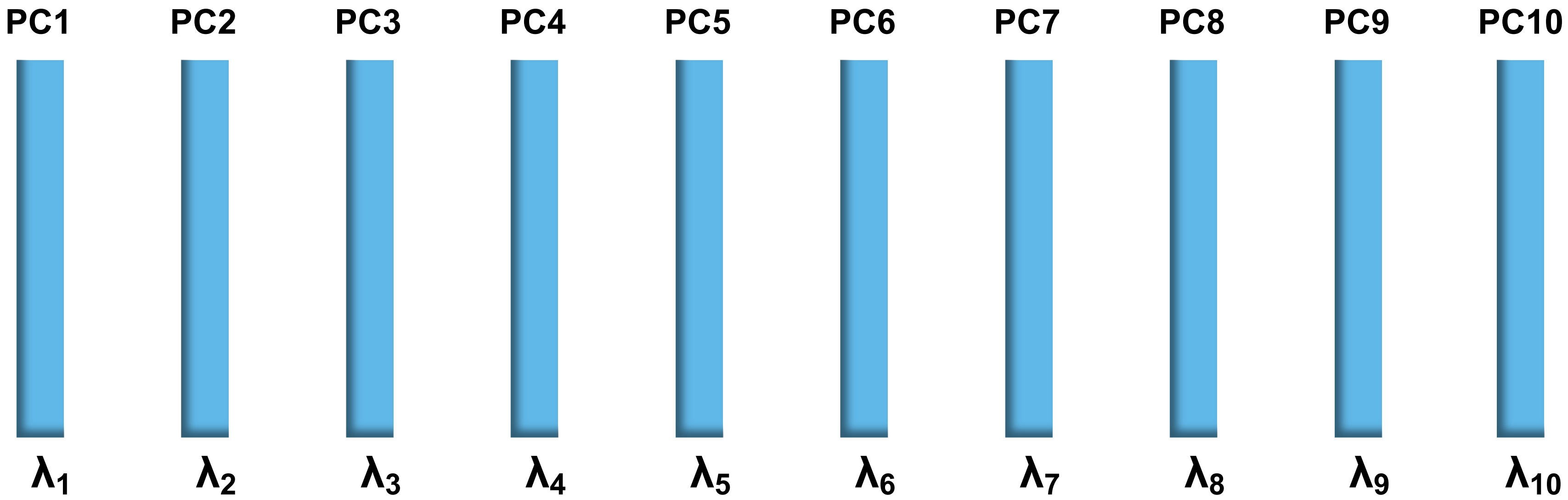

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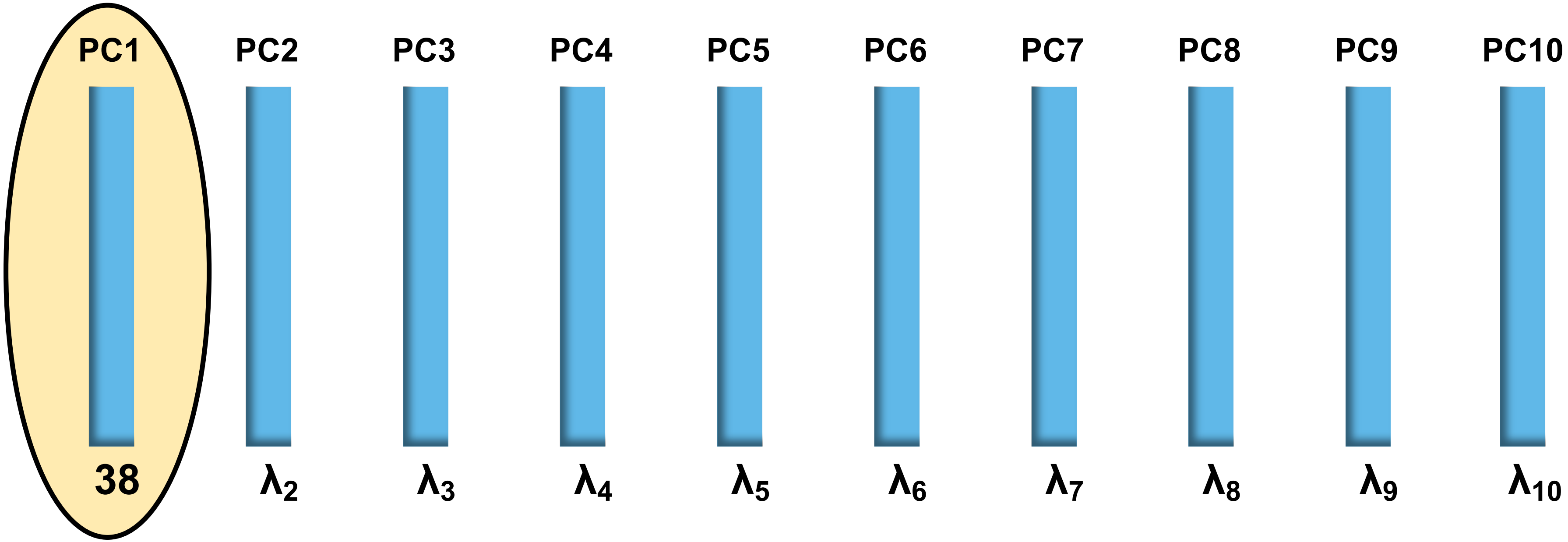

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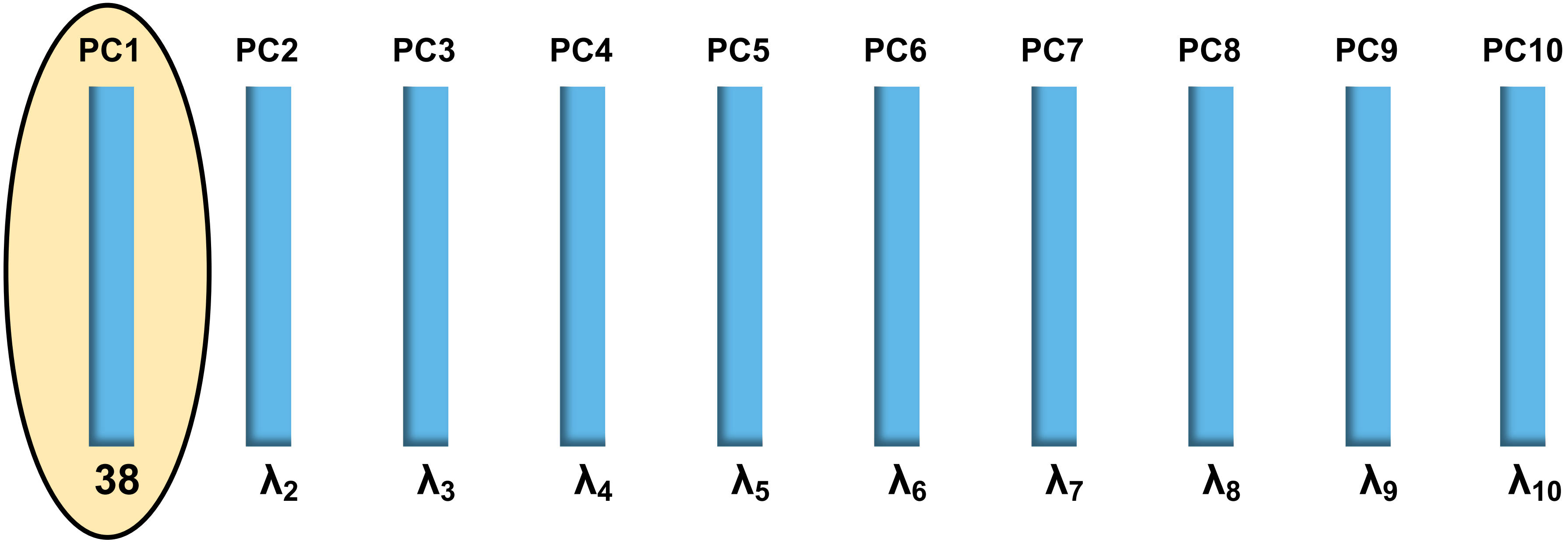
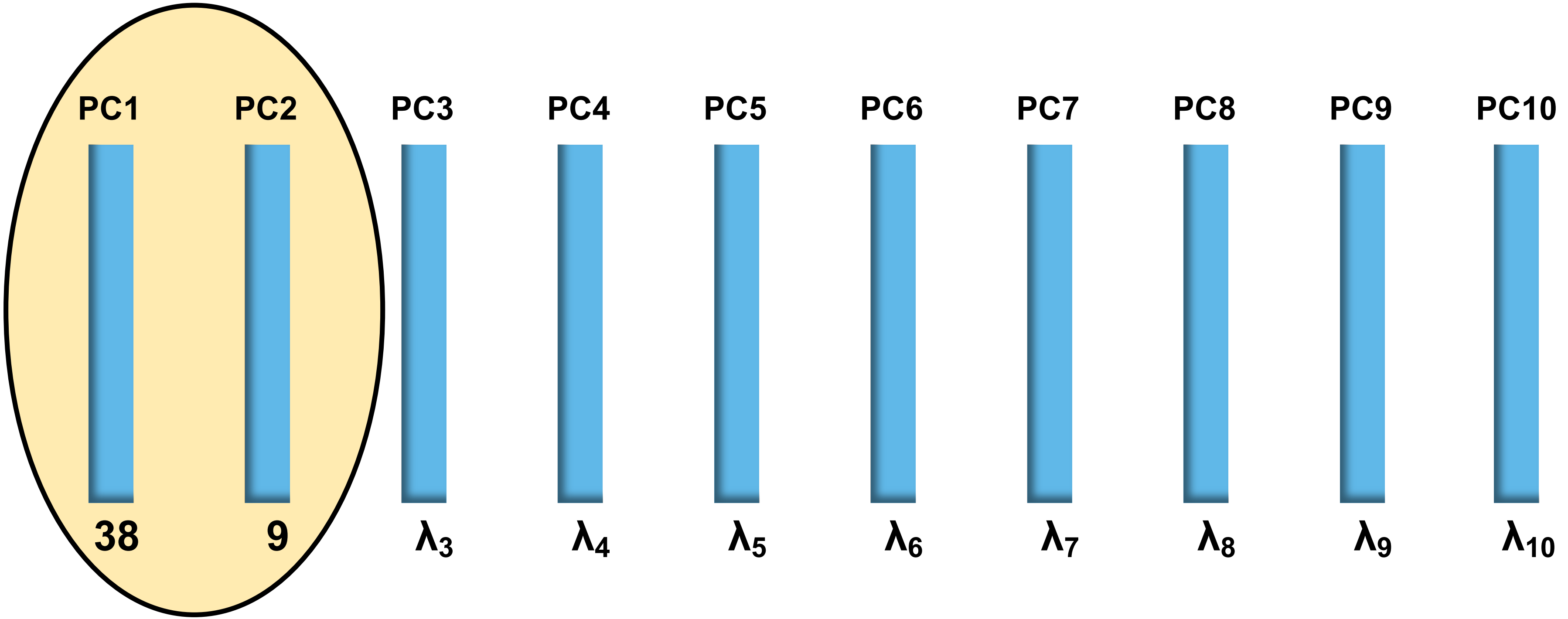


66%

PCA in Higher Dimensions



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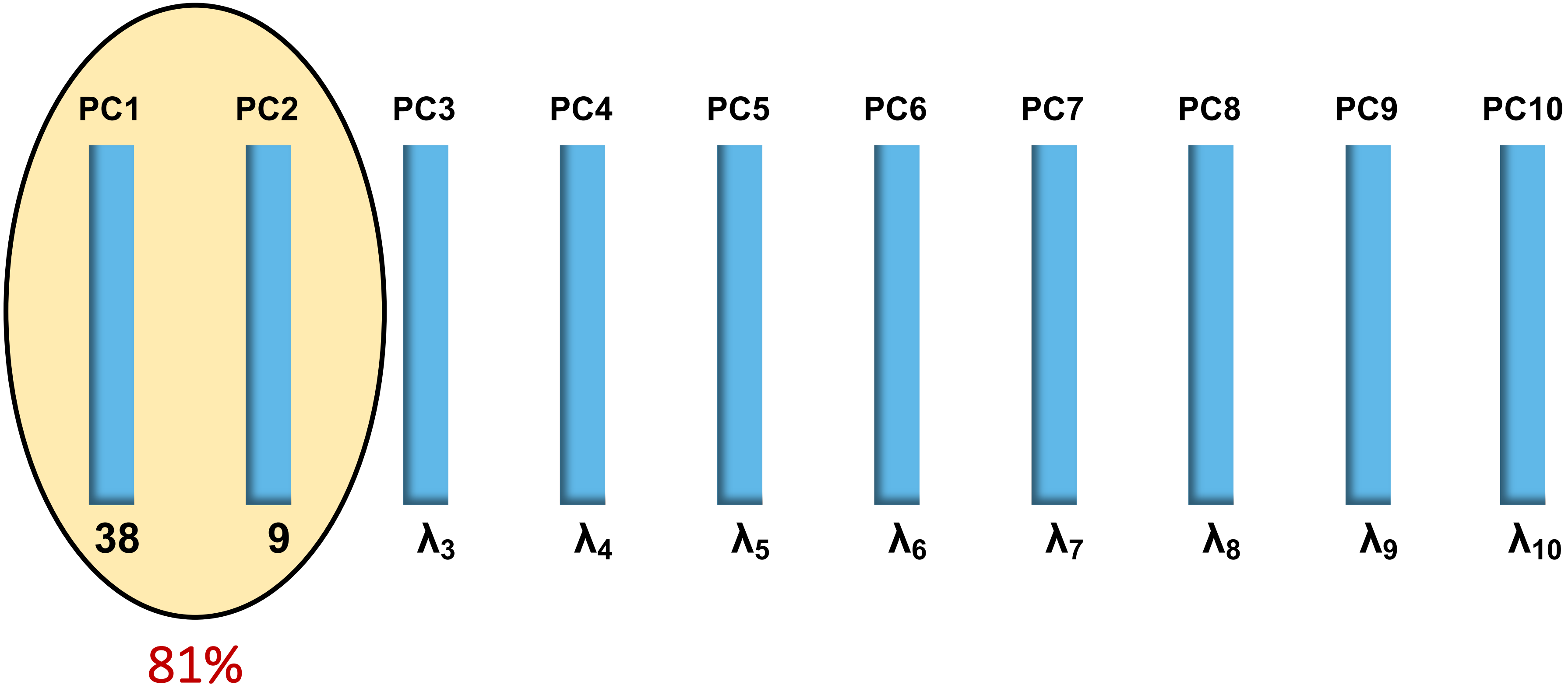

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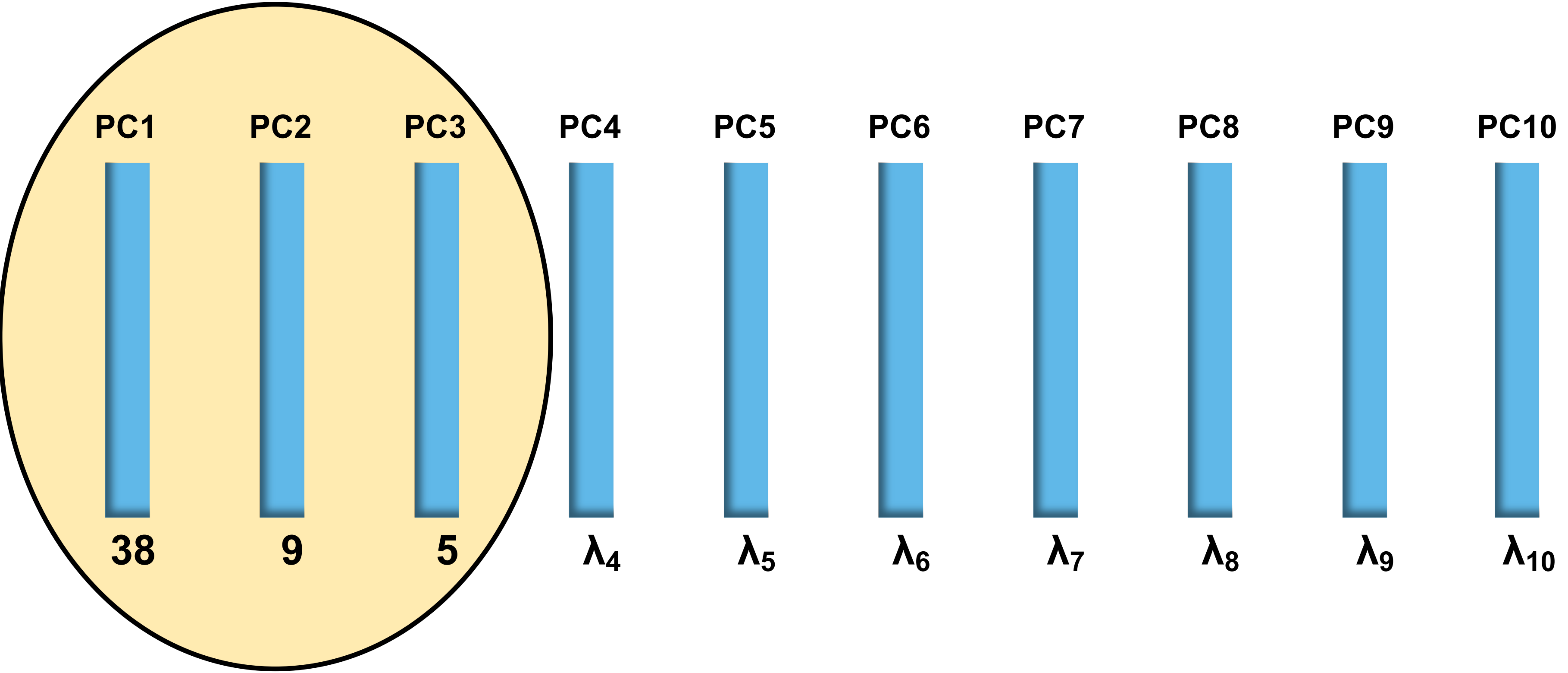

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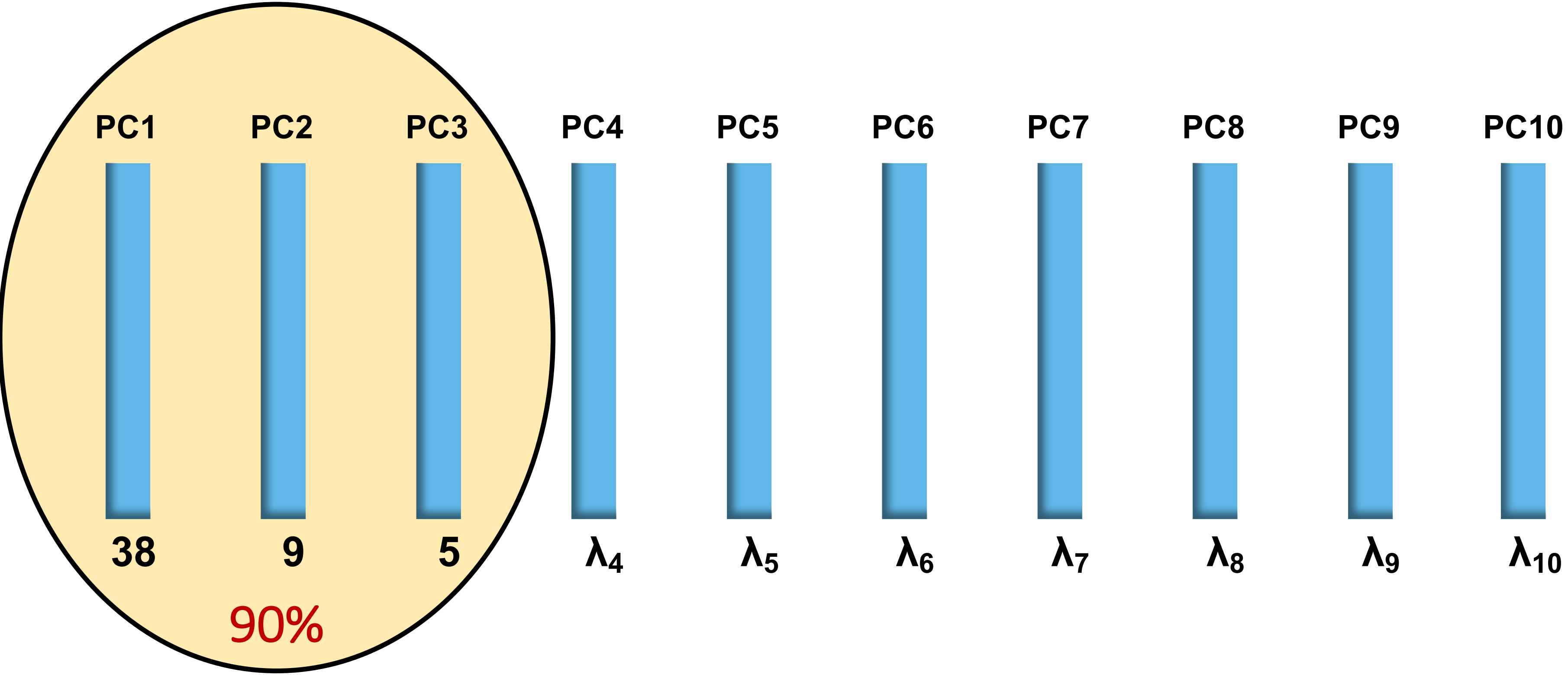

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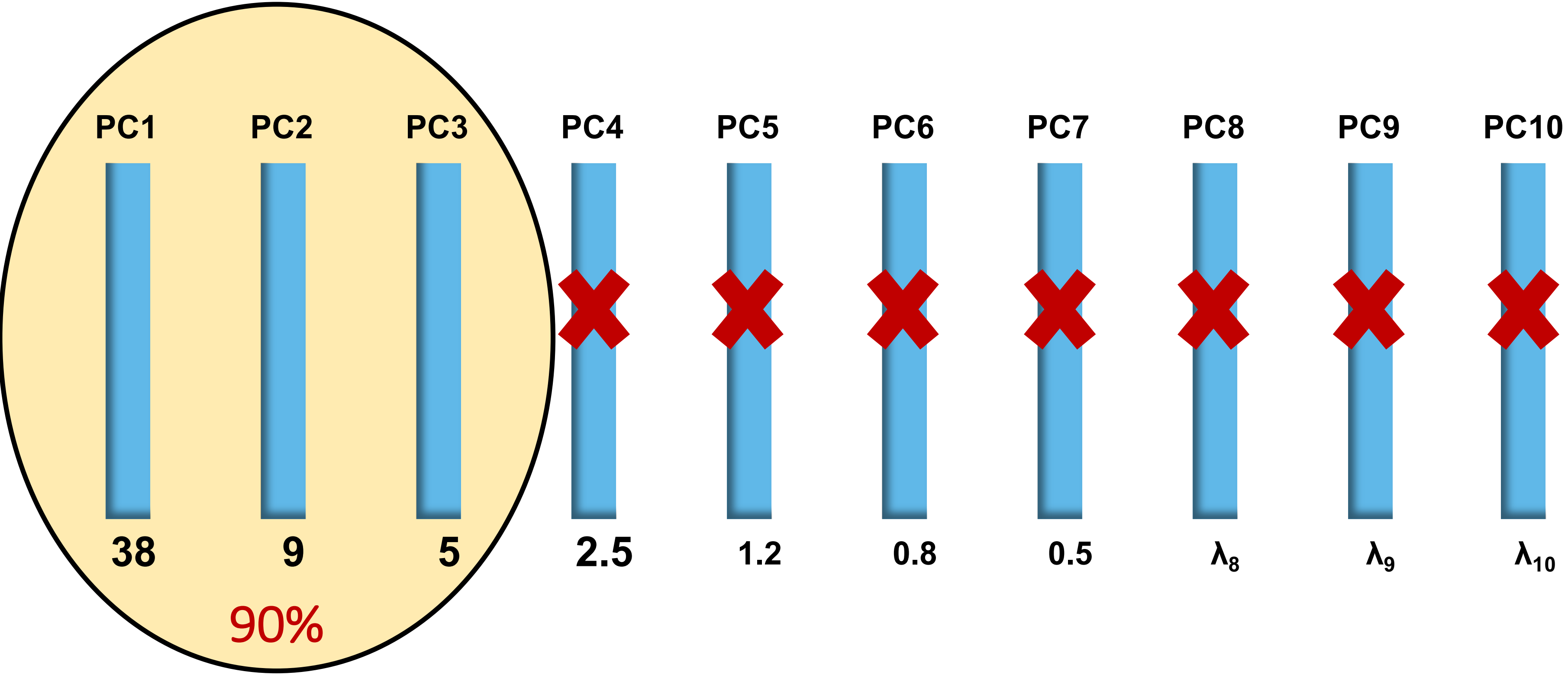

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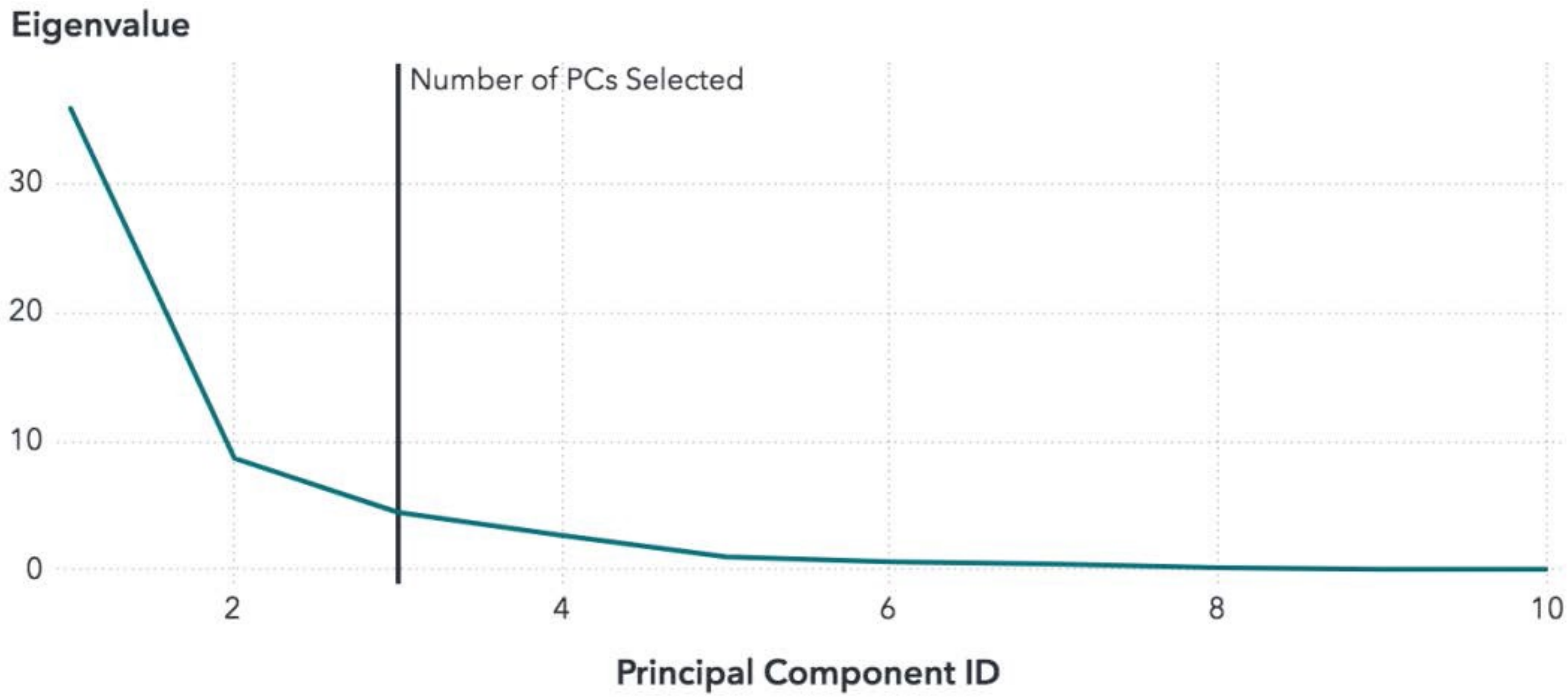
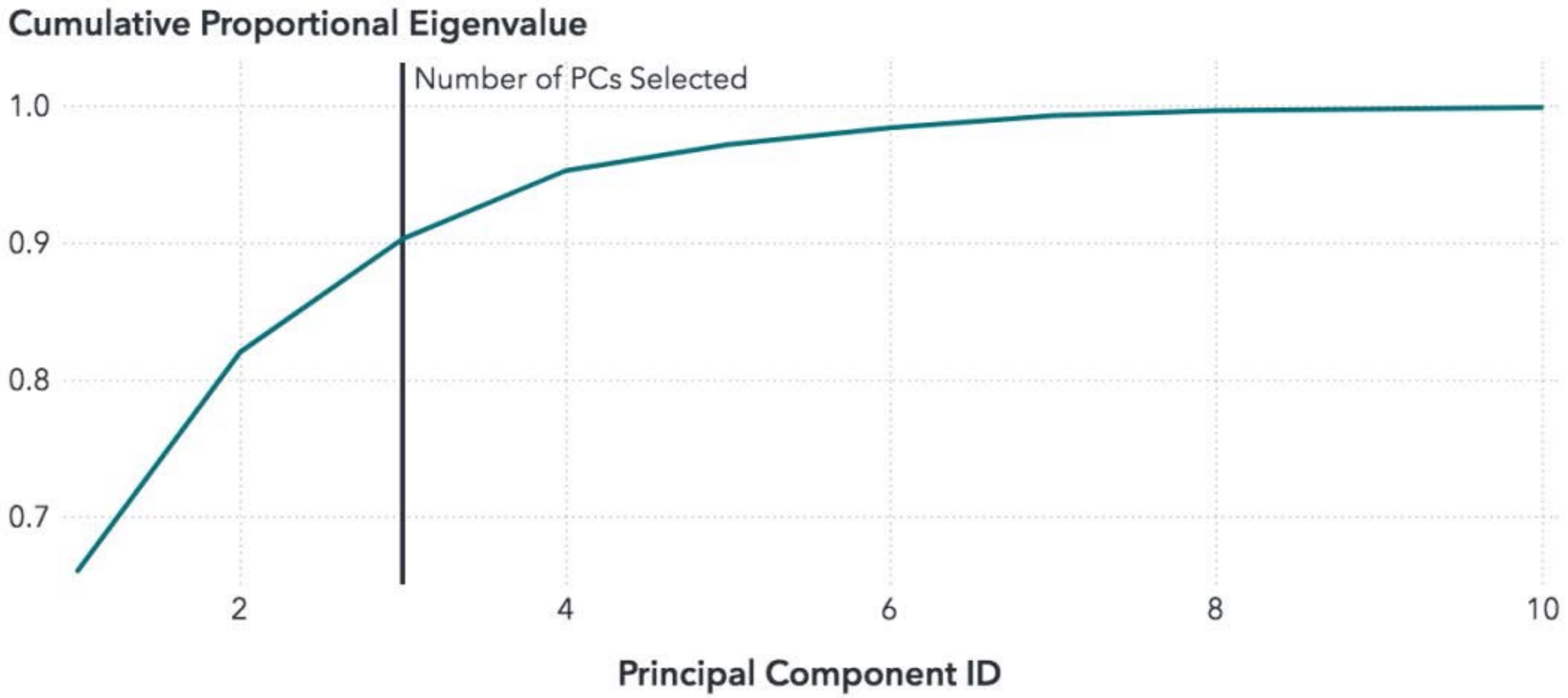
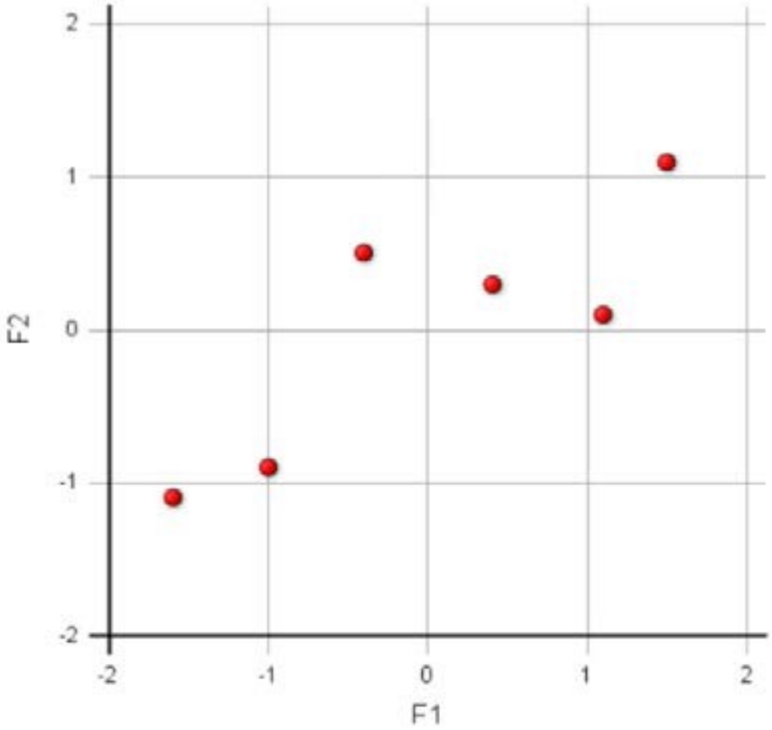


Principal Component Analysis



Reconstruction Error



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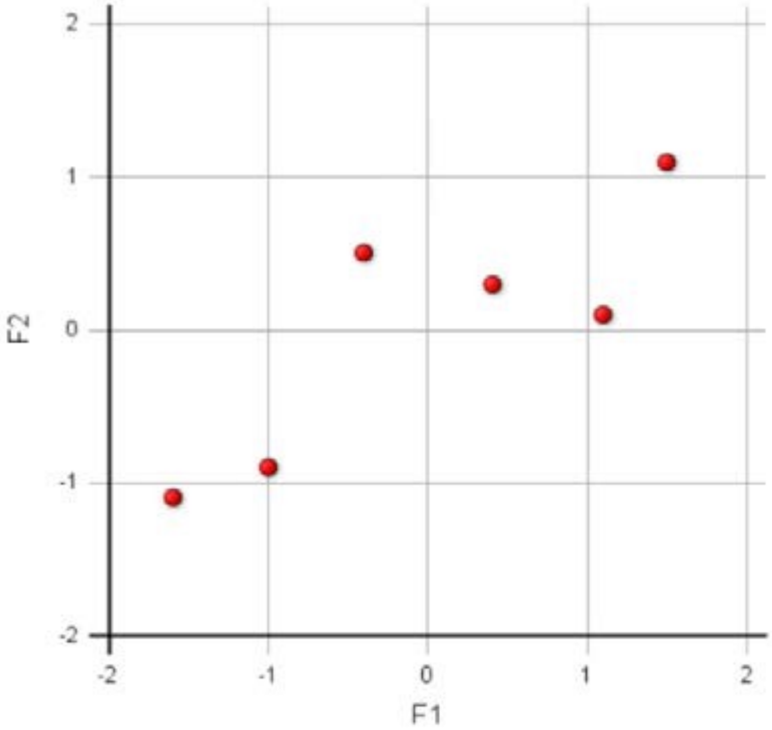

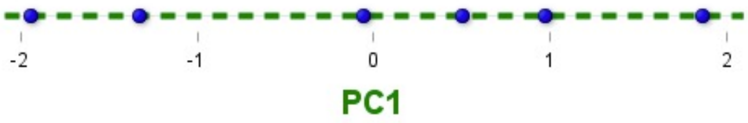

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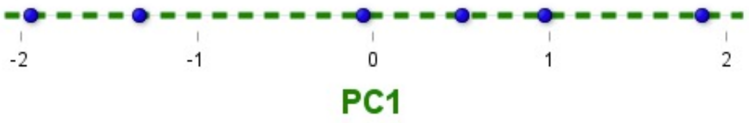

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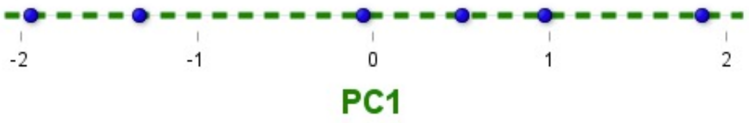

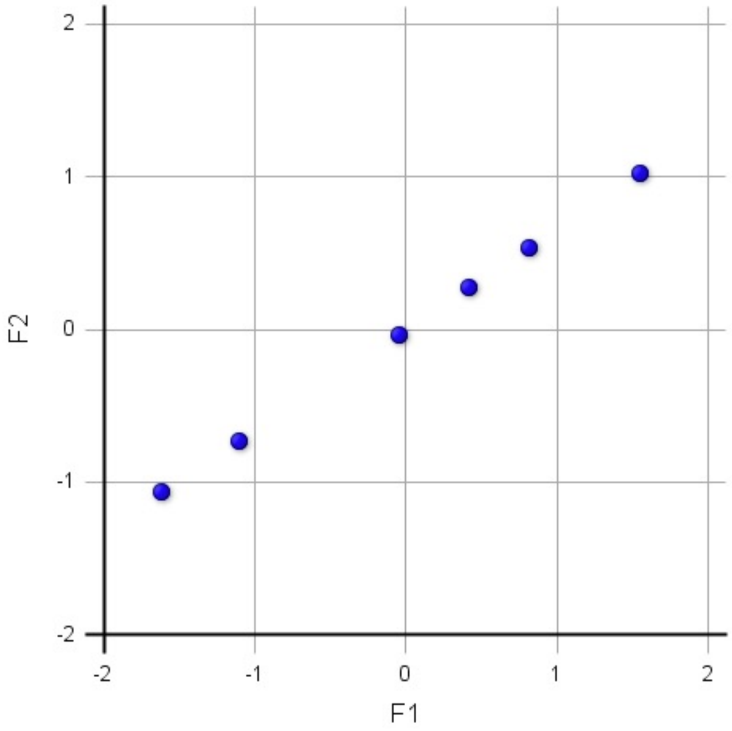

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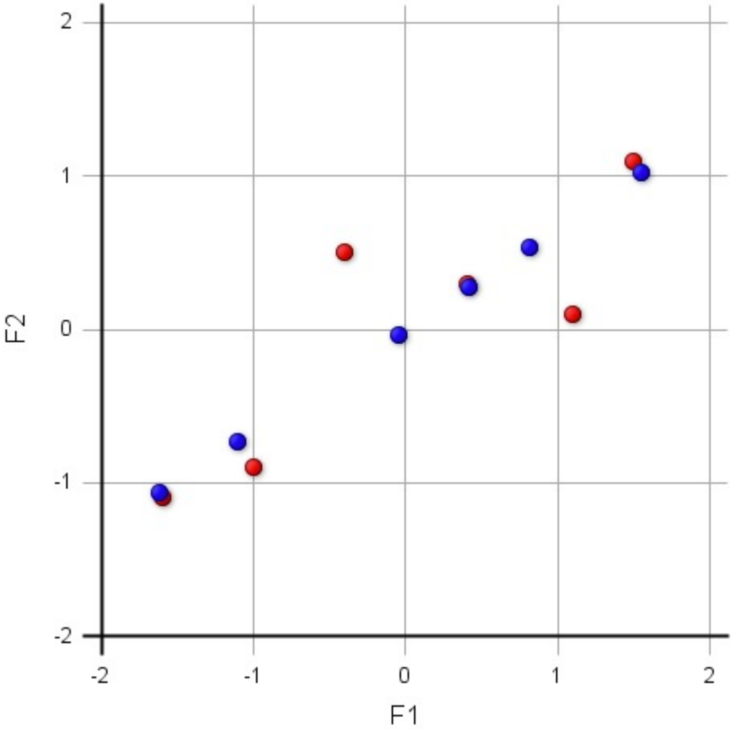

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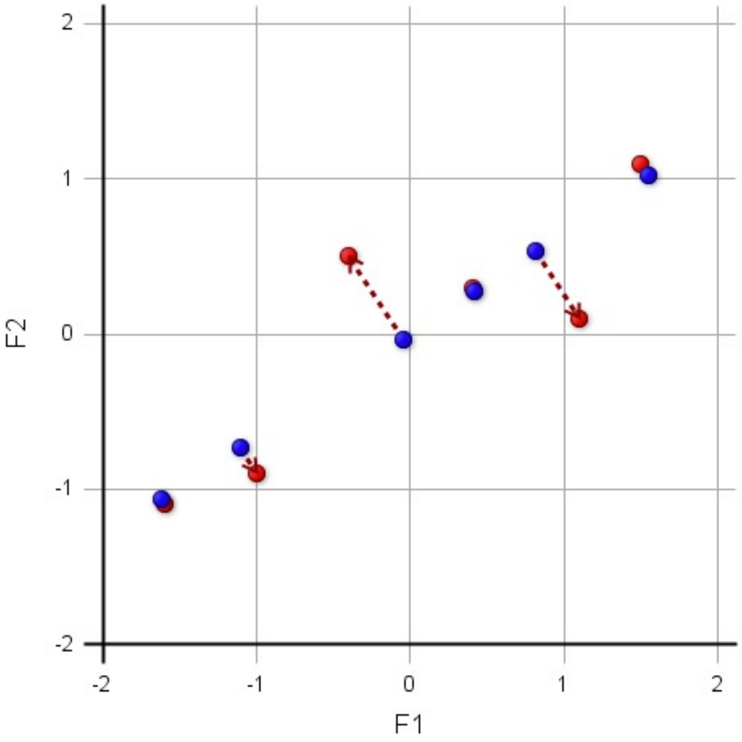

Summary

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PCA finds the new feature set which retains maximum variance.

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Further Reading

- Jolliffe, I.T. 2002. *Principal Component Analysis*, 2nd ed. New York, NY: Springer.

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- Jolliffe, I.T. 2002. *Principal Component Analysis*, 2nd ed. New York, NY: Springer.

- Shlens, Jonathon. 2014 “A Tutorial on Principal Component Analysis” https://arxiv.org/pdf/1404.1100.pdf
- Wiskott, Laurenz. 2013 “Lecture Notes on Principal Component Analysis” http://cs233.stanford.edu/ReferencedPapers/LectureNotes-PCA.pdf

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- Wiskott, Laurenz. 2013 "Lecture Notes on Principal Component Analysis" http://cs233.stanford.edu/ReferencedPapers/LectureNotes-PCA.pdf
- "Principal Component Analysis Explained Visually" by Victor Powell and Lewis Lehe http://setosa.io/ev/principal-component-analysis/

Thank you!

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